

LESSON #3: KITES AND COORDINATE "NOT" PROOFS

Do Now:

Properties of a Kite: Two pairs of consecutive congruent sides.
 Diagonals are perpendicular.

To prove a quadrilateral is a Kite: 4 distance formulas

1. Prove that Quadrilateral ABCD is a kite.

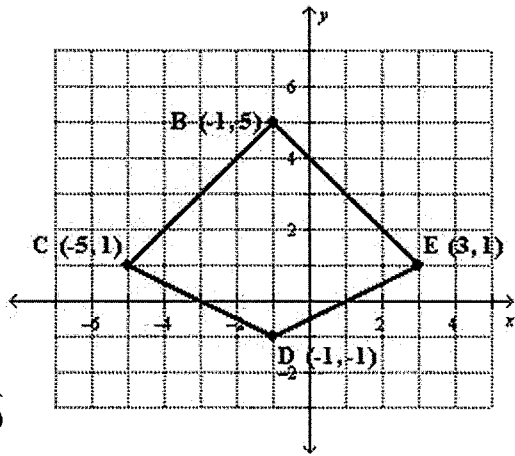
$$\overline{BC} = \sqrt{(-1 - -5)^2 + (5 - 1)^2} = \sqrt{32} \quad \checkmark$$

$$\overline{BE} = \sqrt{(-1 - 3)^2 + (5 - 1)^2} = \sqrt{32}$$

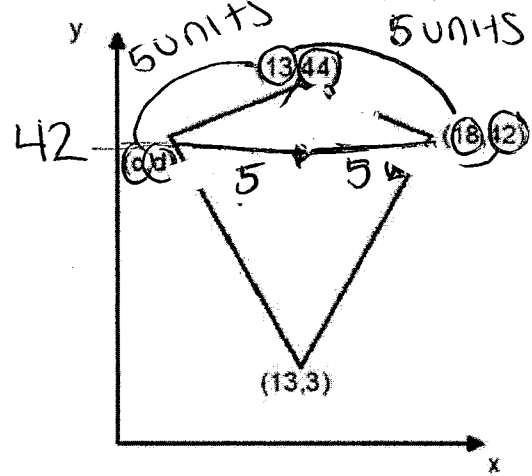
$$\overline{CD} = \sqrt{(-5 - -1)^2 + (1 - -1)^2} = \sqrt{20} \quad \checkmark$$

$$\overline{ED} = \sqrt{(3 - -1)^2 + (1 - -1)^2} = \sqrt{20}$$

∴ Quad ABCD is a kite b/c it has 2 pairs of consecutive \cong sides.



2. Determine the values of (c, d).



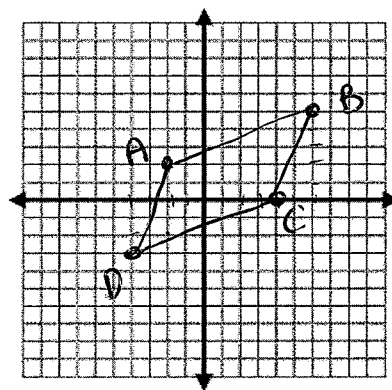
$$c = 13 - 5 = 8$$

$$d = 42$$

(8, 42)

HOW DO WE COMPLETE COORDINATE "NOT" PROOFS?!?!?

3. Given the coordinates of Quadrilateral $ABCD$ are $A(-2,2), B(6,5), C(4,0), D(-4,-3)$ Prove that quadrilateral $ABCD$ is a parallelogram, but not a rectangle.



Step 1: Prove \square (Diagonals bisect)

$$MP_{AC} = \left(\frac{-2+4}{2}, \frac{2+0}{2} \right) = \left(\frac{2}{2}, \frac{2}{2} \right) = (1, 1)$$

$$MP_{BD} = \left(\frac{6+(-4)}{2}, \frac{5+(-3)}{2} \right) = \left(\frac{2}{2}, \frac{2}{2} \right) = (1, 1)$$

$\therefore ABCD$ is a \square b/c the diagonals bisect.

Step 2: Prove diagonals are NOT \cong .

$$d_{\overline{AC}} = \sqrt{(4-(-2))^2 + (0-2)^2} = \sqrt{40}$$

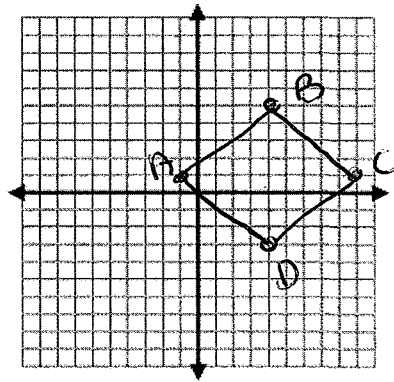
$$d_{\overline{BD}} = \sqrt{(-4-6)^2 + (-3-5)^2} = \sqrt{164}$$

$\therefore ABCD$ is NOT a rectangle b/c diagonals $\overline{AC} \neq \overline{BD}$

4. Given the coordinates of Quadrilateral $ABCD$ are $A(-1,1)$, $B(4,5)$, $C(9,1)$, $D(4,-3)$. Prove that Quadrilateral $ABCD$ is a rhombus, but **not** a square.

Plan:

- ① prove diagonals are \perp
- ② prove diagonals are **NOT** \cong



$$\textcircled{1} m_{\overline{AC}} = \frac{1-1}{9-1} = \frac{0}{8} = 0$$

★ the reciprocal of $\frac{0}{1}$ is $\frac{1}{0}$ (undefined!)

$$m_{\overline{BD}} = \frac{-3-5}{4-4} = \frac{-8}{0} = \text{undefined}$$

$\therefore ABCD$ is a rhombus b/c diagonals are \perp

$\textcircled{2}$

$$d_{\overline{AC}} = \sqrt{(9-1)^2 + (1-1)^2} = \sqrt{100} = 10$$

$$d_{\overline{BD}} = \sqrt{(-3-5)^2 + (4-4)^2} = \sqrt{64} = 8$$

$\therefore ABCD$ is NOT a square b/c $\overline{AC} \neq \overline{BD}$ (diagonals are not \cong)

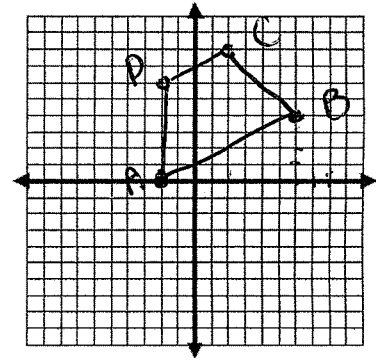
5.

Quadrilateral $ABCD$ has coordinates $A(-2,0)$, $B(6,4)$, $C(2,8)$, and $D(-2,6)$.

Using coordinate geometry, prove that

a $ABCD$ is a trapezoid - one pair opp sides \parallel

b $ABCD$ is not an isosceles trapezoid - $AD \not\cong BC$



$$\textcircled{1} m_{\overline{DC}} = \frac{6-8}{-2-2} = \frac{-2}{-4} = \frac{1}{2}$$

✓ $\overline{AB} \parallel \overline{DC}$

$$m_{\overline{AB}} = \frac{4-0}{6-2} = \frac{4}{8} = \frac{1}{2}$$

\therefore $ABCD$ is a trapezoid b/c at least one pair of opp. sides are \parallel

$$\textcircled{2} d_{\overline{AD}} = \sqrt{(-2-2)^2 + (6-0)^2} = \sqrt{36} = 6$$

$\overline{AD} \not\cong \overline{BC}$

$$d_{\overline{BC}} = \sqrt{(8-4)^2 + (2-6)^2} = \sqrt{32}$$

\therefore $ABCD$ is NOT an isosceles trapezoid b/c the legs are not \cong