Name: Key

Date: 4/30/18

CC GEOMETRY

TROICI

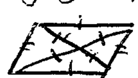
LESSON #7: COORDINATE PROOFS

QUADRILATERAL

A polygon w/ 4 sides

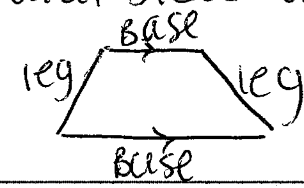
PARALLELOGRAM ▱

① opposite sides are ≅
② opposite sides are ∥
③ opposite ∠'s are ≅
④ consecutive ∠'s are supplementary
⑤ Diagonals bisect each other



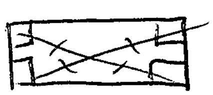
TRAPEZOID

At least one pair opposite sides are ∥ (Bases)
Non parallel sides are called legs



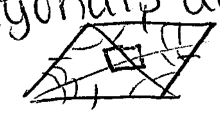
RECTANGLE

all properties of ▱ +
① Diagonals are ≅
② 4 right ∠'s



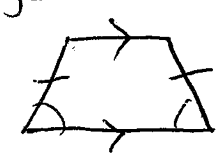
RHOMBUS

all properties of ▱ +
① All sides are ≅
② diagonals bisect ∠'s
③ Diagonals are ⊥



ISOSCELES TRAPEZOID

A trapezoid w/ 2 ≅ legs + base ∠'s



SQUARE

All properties of
- parallelograms
- Rectangles
- Rhombi

METHODS TO PROVE QUADRILATERALS:

1. TO PROVE A QUADRILATERAL IS A **PARALLEOGRAM:**

WITH WORDS	WITH COORDINATE GEOMETRY
- 2 sets of opp. sides are ≅ - 2 sets of opp. sides are ∥ - 1 set of opp. sides are ∥ and ≅ - Diagonals bisect	- Distance 4x (2 sets of = sides) - slope 4x (2 sets of = slopes) - 2 Distance + 2 slope - midpoint 2x (equal mp's)

#'s 4-7

2. TO PROVE A QUADRILATERAL IS A **RECTANGLE:**

WITH WORDS	WITH COORDINATE GEOMETRY
① A ▱ with 1 right ∠	① midpoint 2x ② slope 2x (opp. reciprocal)
② A ▱ with ≅ diagonals	① midpoint 2x ② Distance 2x (= lengths)

#'s 8 and 9

3. TO PROVE A QUADRILATERAL IS A **RHOMBUS:**

WITH WORDS	WITH COORDINATE GEOMETRY
① prove ▱ (diagonals bisect) ② prove Diagonals are ⊥	① midpoint 2x ② slope 2x (opp. reciprocal slopes)

#'s 10 and 11

4. TO PROVE A QUADRILATERAL IS A **SQUARE:**

WITH WORDS	WITH COORDINATE GEOMETRY
① Prove ▱ (diagonals bisect) ② prove rectangle (diagonals are ≅) ③ Prove 4 ≅ sides	① midpoint 2x ② Distance 2x ③ Distance 4x

#'s 12-14

5. TO PROVE A QUADRILATERAL IS A **TRAPEZOID**:

WITH WORDS	WITH COORDINATE GEOMETRY
At least one pair of opposite sides are ∥	slope formula 2x need EQUAL slopes

#1st and 2

6. TO PROVE A QUADRILATERAL IS AN **ISOSCELES TRAPEZOID**:

WITH WORDS	WITH COORDINATE GEOMETRY
- At least one pair of opp. sides are ∥ - The non-∥ sides are ≅	- slope formula 2x (equal slopes) - Distance 2x

#3

7. TO PROVE A POLYGON IS A **RIGHT TRIANGLE**:

WITH WORDS	WITH COORDINATE GEOMETRY
Prove 1 right ∡	slope 3x (opp. reciprocal slopes)

8. TO PROVE A POLYGON IS AN **ISOSCELES TRIANGLE**:

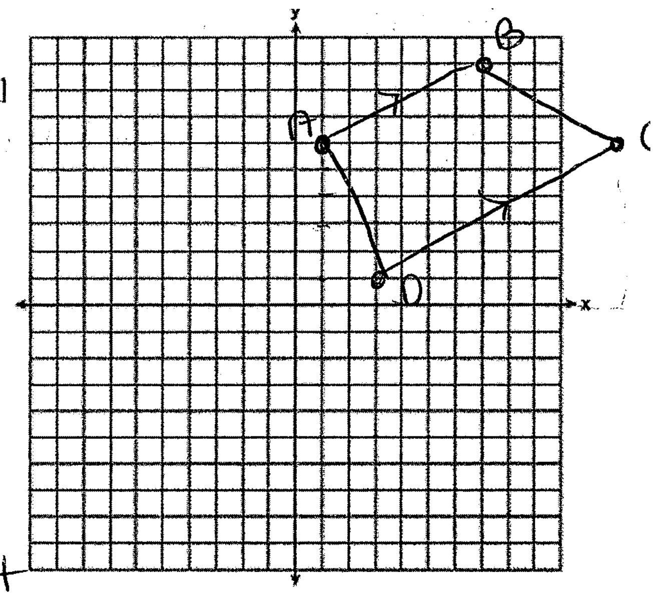
WITH WORDS	WITH COORDINATE GEOMETRY
Prove 2 ≅ sides	Distance 3x (2 ≅)

9. TO PROVE A POLYGON IS AN **EQUILATERAL TRIANGLE**:

WITH WORDS	WITH COORDINATE GEOMETRY
Prove 3 ≅ sides	Distance 3x (3 ≅)

	DISTANCE	SLOPE	MIDPOINT
FORMULA	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	$m=\frac{y_2-y_1}{x_2-x_1}$	$mp=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
KEY WORDS	congruent, equal	parallel or perpendicular	Bisect

1. Given: $A\overset{x_1\,y_1}{(1,6)}$, $B\overset{x_2\,y_2}{(7,9)}$, $C\overset{x_1\,y_1}{(13,6)}$, and $D\overset{x_2\,y_2}{(3,1)}$
Prove: *ABCD* is a trapezoid. [The use of the coordinate plane is optional]



$m_{\overline{AB}} = \frac{9-6}{7-1} = \frac{3}{6} = \frac{1}{2}$

$m_{\overline{DC}} = \frac{1-6}{3-13} = \frac{-5}{-10} = \frac{1}{2}$

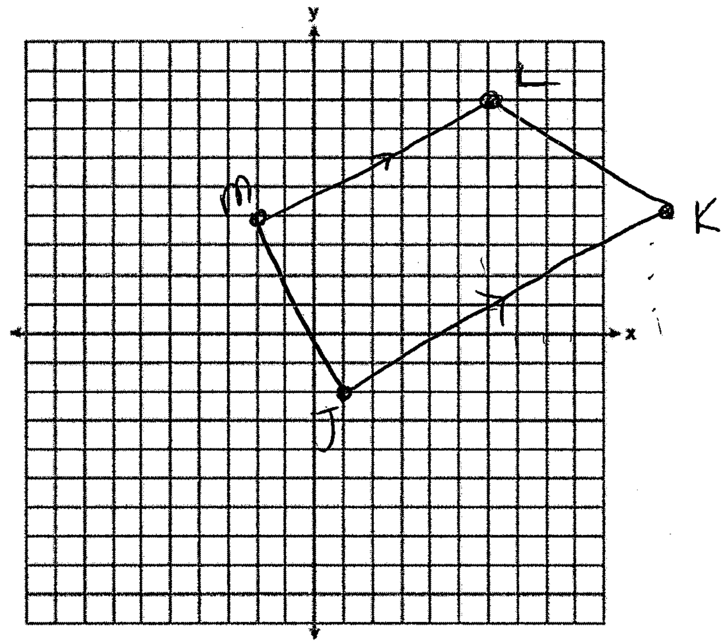
∴ ABCD is a trapezoid b/c at least one pair of opposite sides are ∥

2. The coordinates of quadrilateral $JKLM$ are $J(1,-2)$, $K(13,4)$, $L(6,8)$, and $M(-2,4)$.
Prove that quadrilateral $JKLM$ is a trapezoid. [*The use of the accompanying grid is optional.*]

$m_{ML} = \frac{4-8}{-2-6} = \frac{-4}{-8} = \frac{1}{2}$

$m_{JK} = \frac{4--2}{13-1} = \frac{6}{12} = \frac{1}{2}$

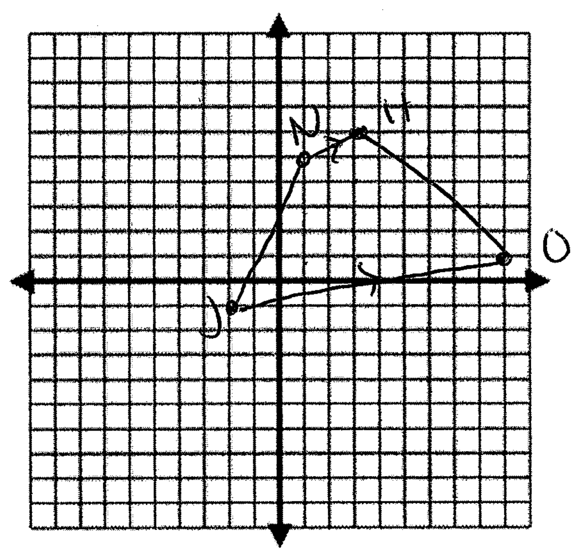
∴ JKLM is a trapezoid b/c at least 1 pair of opposite sides are ∥.



3. Given the coordinates of Quadrilateral $JOHN$ are $J(0,-2)$, $O(9,1)$, $H(4,6)$, $N(1,5)$.
Prove that Quadrilateral $JOHN$ is an Isosceles Trapezoid.

$m_{\overline{NH}} = \frac{5-6}{1-4} = \frac{-1}{-3} = \frac{1}{3}$ ✓

$m_{\overline{JO}} = \frac{1--2}{9-0} = \frac{3}{9} = \frac{1}{3}$

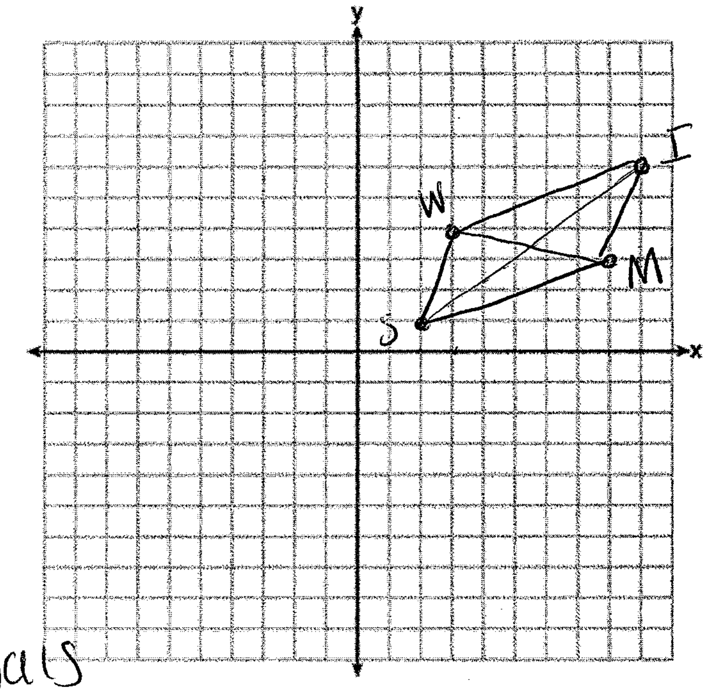


$d_{\overline{JN}} = \sqrt{(1-0)^2 + (5-2)^2} = \sqrt{50}$ ✓

$d_{\overline{HO}} = \sqrt{(4-9)^2 + (6-1)^2} = \sqrt{50}$

∴ JOHN is a trapezoid b/c at least 1 pair of sides are ∥
JOHN is an isosceles trapezoid b/c the non-parallel sides are ≅.

4. Prove that quadrilateral SWIM is a parallelogram:
S(2,1) W(3,4) I(9,6) M(8,3)



$mp_{\overline{SI}} = \left(\frac{2+9}{2}, \frac{1+6}{2}\right) = \left(\frac{11}{2}, \frac{7}{2}\right)$

$\boxed{(5.5, 3.5)}$

$mp_{\overline{WM}} = \left(\frac{3+8}{2}, \frac{4+3}{2}\right) = \left(\frac{11}{2}, \frac{7}{2}\right)$

$= \boxed{(5.5, 3.5)}$

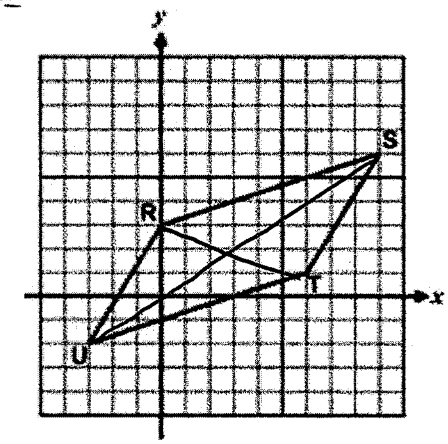
∴ SWIM is a ▱ b/c the diagonals bisect each other

5. On the diagram quadrilateral *RSTU* is shown with vertices $R(0,3)$, $S(9,6)$, $T(6,1)$ and $U(-3,-2)$. Prove that *RSTU* is a parallelogram.

$mp_{\overline{RT}} = \left(\frac{0+6}{2}, \frac{3+1}{2}\right) = (3,2)$

$mp_{\overline{US}} = \left(\frac{9+-3}{2}, \frac{6+-2}{2}\right) = (3,2)$

$\therefore$ RSTU is a ▱ b/c the diagonals bisect each other



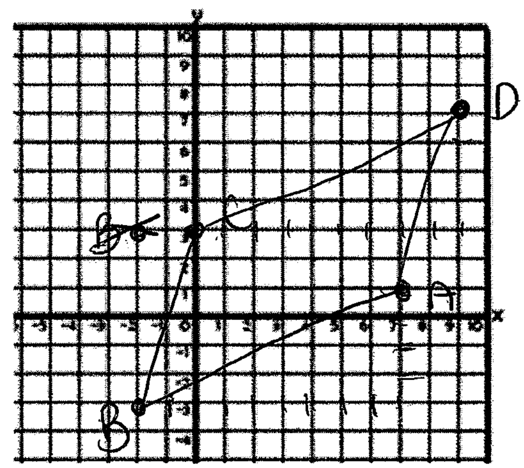
6. Parallelogram *ABCD* has coordinates $A(7,1)$, $B(-2,-3)$ and $C(0,3)$. What must be the coordinates of point D? Explain how you arrived at your answer.

$D = (9,7)$

Slope of $\overline{Ba} = \frac{4}{9}$ (count boxes)

the slope of CD must also be $\frac{4}{9}$

making D (9,7).

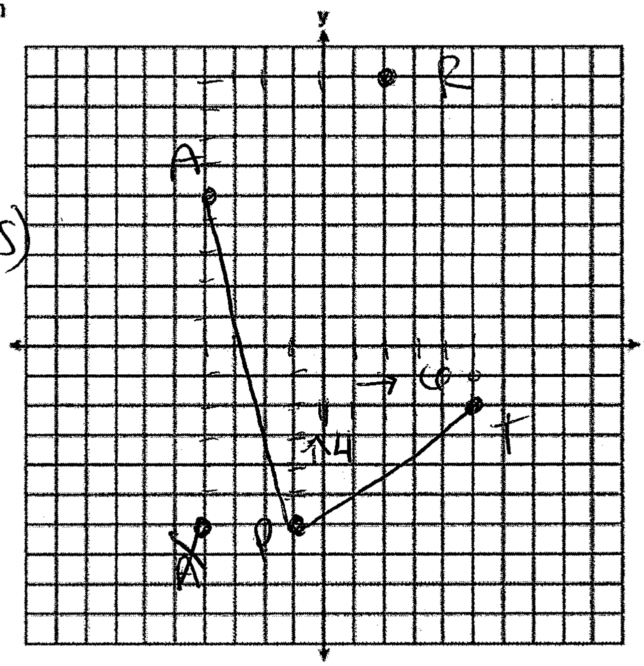


7. In the coordinate plane, the vertices of triangle PAT are $P(-1,-6)$, $A(-4,5)$ and $T(5,-2)$. State the coordinates of R such that $PART$ is a parallelogram

R (2,9)

slope of $\overline{PT} = \frac{4}{6}$ (count boxes)

∴ slope of AR must also be $\frac{4}{6}$



8. Triangle KLM has vertices $K(0,4)$, $L(4,2)$, $M(1,-4)$. [The use of the axes is optional]

a) State the coordinates of N that will make KLMN a rectangle.

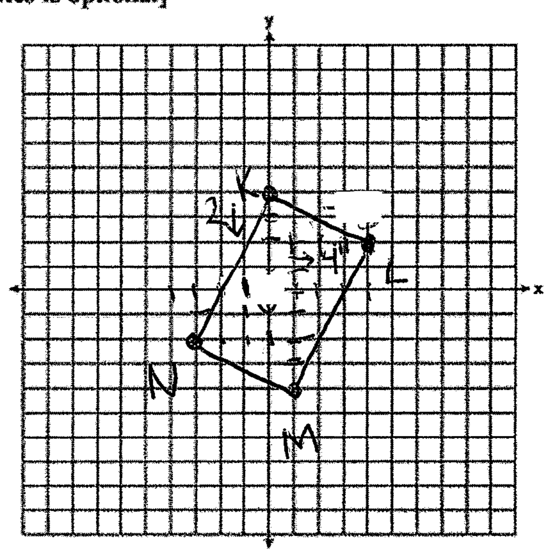
N (-3,-2)

b) Prove that KLMN is a rectangle.

① prove ▱

$mp_{\overline{KM}} = \left(\frac{0+1}{2}, \frac{4+-4}{2}\right) = (.5,0)$ ✓

$mp_{\overline{LN}} = \left(\frac{4+-3}{2}, \frac{2+-2}{2}\right) = (.5,0)$



② prove diagonals are ≅

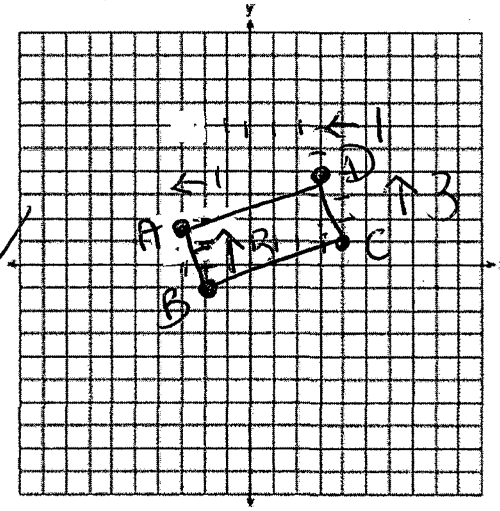
$d_{KM} = \sqrt{(1-0)^2+(-4-4)^2} = \sqrt{65}$ ✓

$d_{LN} = \sqrt{(-3-4)^2+(-2-2)^2} = \sqrt{65}$

∴ KLMN is a ▱ b/c diagonals bisect.
KLMN is a rectangle b/c diagonals are ≅

9. The coordinates of the vertices of triangle BCD are $B(-2,-1)$, $C(4,1)$, and $D(3,4)$.
a) State the coordinates of A that will make ABCD a rectangle. [The use of the axes is optional]

$(-3,2) = A$



b) Prove that ABCD is a rectangle.

① prove ▱

$mp_{\overline{AC}} = \left(\frac{-3+4}{2}, \frac{2+1}{2}\right) = (.5, 1.5)$

$mp_{\overline{BD}} = \left(\frac{-2+3}{2}, \frac{-1+4}{2}\right) = (.5, 1.5)$

② prove diagonals are ≅

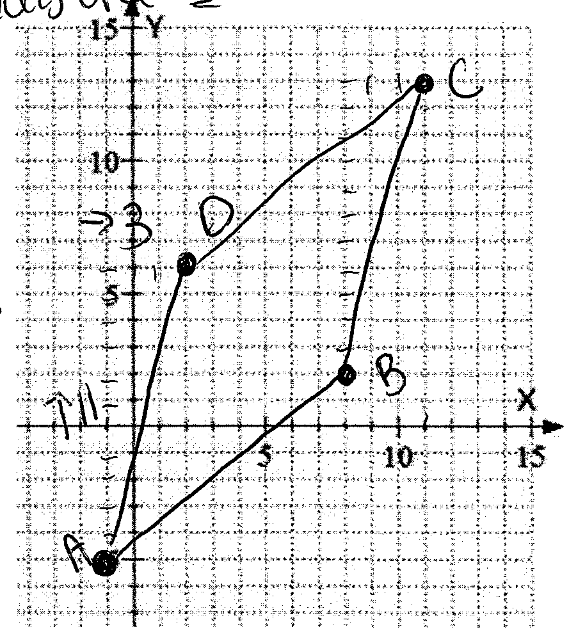
$d_{\overline{AC}} = \sqrt{(-3-4)^2 + (2-1)^2} = \sqrt{50}$

$d_{\overline{BD}} = \sqrt{(-2-3)^2 + (-1-4)^2} = \sqrt{50}$

Quad ABCD is a ▱ b/c the diagonals bisect.
Quad ABCD is a rectangle b/c diagonals are ≅

10. Triangle ABC has coordinates of $A(-1,-5)$, $B(8,2)$, $C(11,13)$.
a) State the coordinates of D that will make ABCD a rhombus.

$(2,6) = D$



b) Prove that ABCD is a rhombus.

① prove ▱

$mp_{\overline{AC}} = \left(\frac{-1+11}{2}, \frac{-5+13}{2}\right) = (5,4)$

$mp_{\overline{BD}} = \left(\frac{8+2}{2}, \frac{2+6}{2}\right) = (5,4)$

② prove diagonals bisect

$m_{\overline{AC}} = \frac{13--5}{11--1} = \frac{18}{12} = \frac{3}{2}$

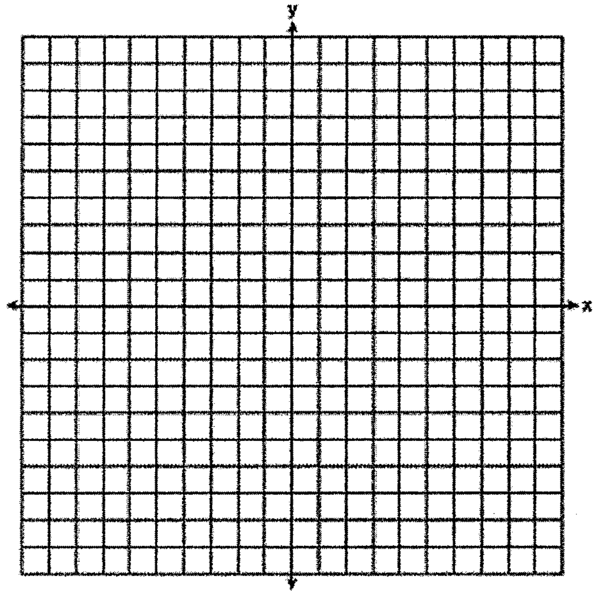
$m_{\overline{BD}} = \frac{6-2}{2-8} = \frac{4}{-6} = -\frac{2}{3}$

∴ $AC \perp BD$

- Quad ABCD is a ▱ b/c the diagonals bisect
- Quad ABCD is a rhombus b/c the diagonals are ⊥

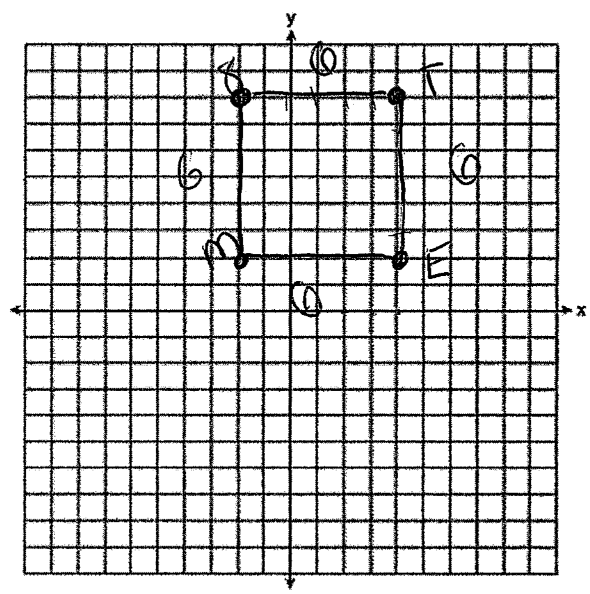
11. Jim is experimenting with a new drawing program on his computer. He created triangle EAM with coordinates, $E(-5,-4)$, $A(2,-1)$, and $M(5,6)$.

a) State the coordinates of T that will make TEAM a rhombus



b) Prove that TEAM is a rhombus.

12. Prove that quadrilateral METS is a square given the vertices $M\ (-2,2), E\ (4,2), T\ (4,8),$ and $S\ (-2,8)$



① Prove ▱

$mp_{\overline{MT}} = \left(\frac{-2+4}{2}, \frac{2+8}{2}\right) = (1,5)$ ✓

$mp_{\overline{ES}} = \left(\frac{4+-2}{2}, \frac{2+8}{2}\right) = (1,5)$

∴ METS is a ▱ b/c diagonals bisect

② Prove rectangle

$d_{\overline{MT}} = \sqrt{(4-2)^2+(8-2)^2} = \sqrt{72}$ ✓

$d_{\overline{ES}} = \sqrt{(-2-4)^2+(8-2)^2} = \sqrt{72}$

∴ METS is a rectangle b/c diagonals are ≅

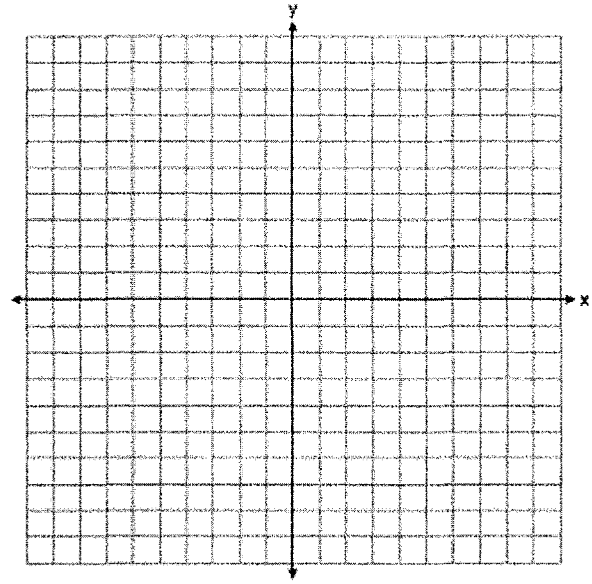
③ Prove square

$m_{\overline{MT}} = \frac{8-2}{4-2} = \frac{6}{6} = 1$ ✓

$m_{\overline{ES}} = \frac{8-2}{-2-4} = \frac{6}{-6} = -1$

∴ METS is a square b/c diagonals are ⊥

13. Square $WXYZ$, the coordinates of W are $(2,-2)$ and the coordinates of Y are $(-4, 2)$. Determine and state the coordinates of vertices X and Z. [The use of the set of axes below is optional.]



14. Square $ABCD$ has vertices $A(-4,2)$ and $C(10,0)$. Determine and state the coordinates of vertices B and D [The use of the set of axes below is optional.]