

Name: Key

Date: 4/10/18

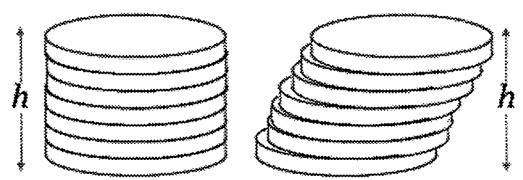
CC GEOMETRY

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**LESSON #9: CAVALERI'S PRINCIPLE**

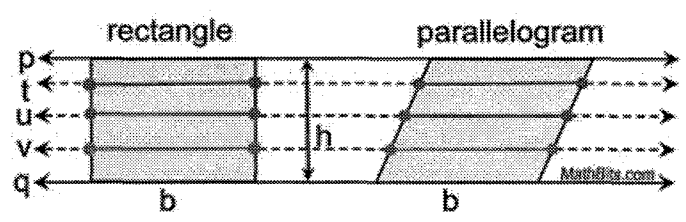
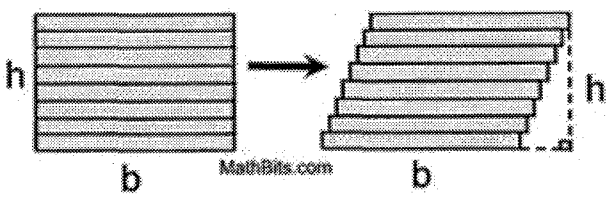
Do Now: The diagram shows two stacks of quarters.

- a) How many quarters are in each stack? 7
- b) Make an assumption about the areas of each quarter.  
each area is the same



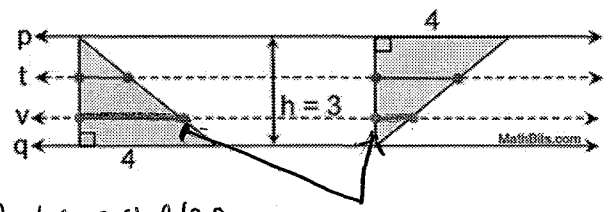
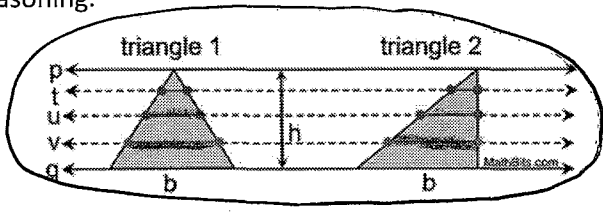
- c) Make an assumption about the volume of each stack of quarters.  
each volume is the same

**CAVALIERI'S PRINCIPLE - 2D**



Suppose two regions in a plane are included between two parallel lines in that plane. If every line parallel to those two lines intersects both regions in the line segments of **congruent lengths**, then the two regions will have **equal areas**.

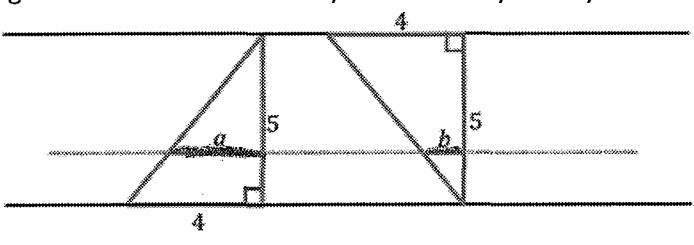
1. Given the two diagrams below, identify which pair of triangles applies Cavalieri's principle. Explain your reasoning.



The 11 lines have equal lengths

Not equal

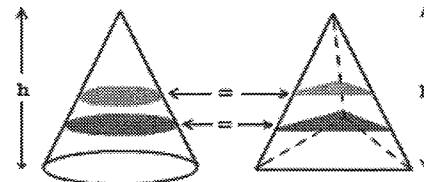
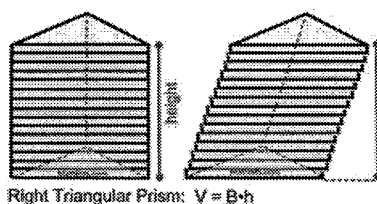
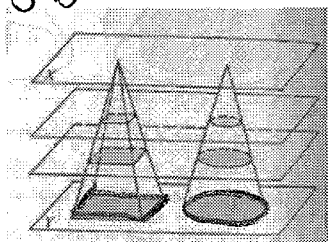
2. Joey says that if two figures have the same height and the same area, then their cross-sectional lengths at each height will be the same. Is Joey correct? Why or why not?



The 11 lines do not have equal lengths, ∴ the cross section areas are not equal.

2D = line = Equal lengths

3D = plane = Equal Areas CAVALIERI'S PRINCIPLE - 3D

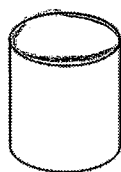
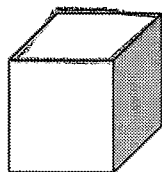


Suppose two regions in three-space are included between two parallel planes. If every plane parallel to these two planes intersects both regions in areas of cross sections are congruent then those two regions have equal volume.

**CRITERIA FOR CAVALIERI'S PRINCIPLE:**

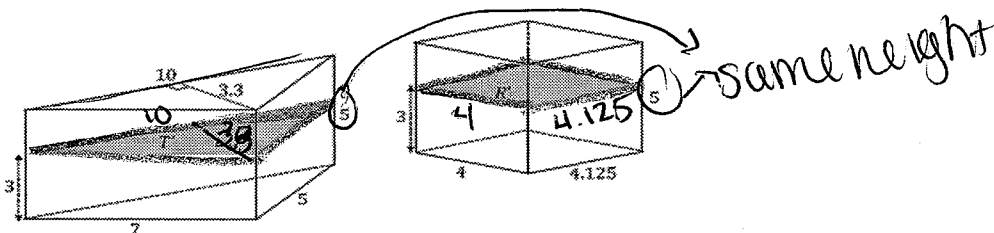
1. Heights must be equal
2. The areas of ~~equal~~ each cross section must be equal at every height.

1. Morgan tells you that Cavalieri's principle cannot be applied to the cylinders shown below because their bases are different. Do you agree or disagree? Explain.



Disagree. If both solids have the same height + the area of the cross sections are equal at every height, then the volumes are equal.

2. The bases of the following triangular prism T and rectangular prism R lie in the same plane. A plane that is parallel to the bases and also a distance 3 from the bottom base intersects both solids and creates cross-sections T' and R'.



a. Find Area(T').

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(10)(3.3)$$

$$A = 16.5 \text{ U}^2$$

c. Find Vol(T).

$$V = Bh$$

$$V = 16.5 \cdot 5$$

$$V = 82.5 \text{ U}^3$$

b. Find Area(R').

$$A = l \cdot w$$

$$A = 4 \cdot 4.125$$

$$A = 16.5 \text{ U}^2$$

d. Find Vol(R).

$$V = l \cdot w \cdot h$$

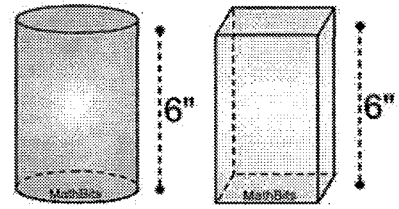
$$V = 4(4.125)(5)$$

$$V = 82.5 \text{ U}^3$$

omg same!

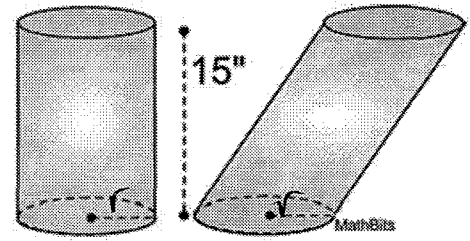
A right circular cylinder and a right rectangular prism are given. True or False? Cavalieri's Principle does not apply to these solids because their bases are not the same shape.

False. If heights are = and cross sectional areas are equal at every height then Cavalieri's principle can be applied



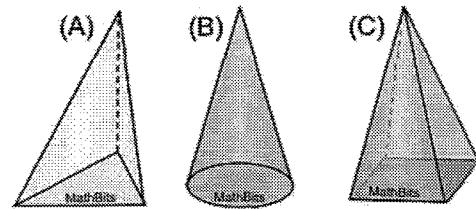
A right circular cylinder and an oblique circular cylinder are given. True or False? If the radii of both cylinders are equal, the volumes of the cylinders will be equal because they have the same height.

True b/c if their radii are = then their cross sectional areas are equal. They also have the same height so it satisfies Cavalieri's principle



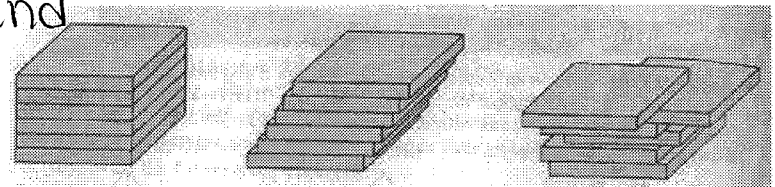
The three solids shown below have the same height and matching cross sectional areas parallel to the bases. Which of the solids have the same volume? Justify your answer.

All 3 b/c it satisfies Cavalieri's principle



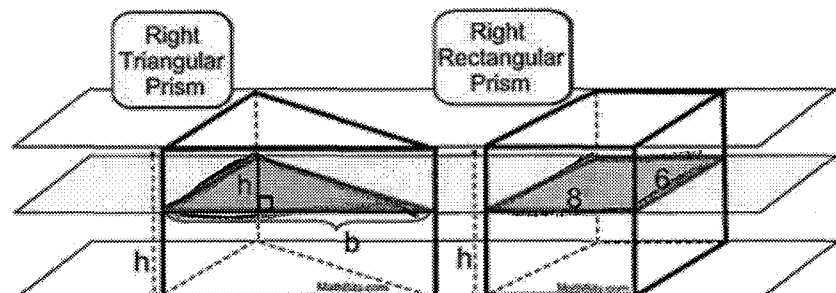
Use Cavalieri's principle to explain why the volumes of these three stacks of books are equal.

Their heights are equal and the areas of each cross section are equal at every height.



By Cavalieri's Principle, this right triangular prism and right rectangular prism have the same volume. If the center plane intersects the solids parallel to their bases, which of the following choices could be the base and height of the triangular cross section?

- a)  $h = 4; b = 12$
- b)  $h = 8; b = 12$**
- c)  $h = 4; b = 8$
- d)  $h = 8; b = 14$



Guess & check!

$$A = \frac{1}{2}bh$$

$$48 = \frac{1}{2}bh$$

$$48 = \frac{1}{2}(8)(12)$$

$$A = l \cdot w$$

$$A = 8 \cdot 6$$

$$A = 48$$

