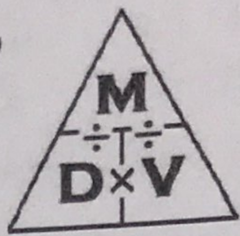
Name: Key
CC GEOMETRY

Date: 3/27/18
TROICI

LESSON #6: APPLICATIONS OF DENSITY

mass	is commonly measured by how much something weighs
volume	is the amount of 3 dimensional space an object occupies (capacity)
Density	how much mass per unit of volume.



Formulas: $D = \frac{m}{V}$ $\quad V = \frac{m}{D}$ $\quad M = D \times V$

**** Be careful with your units****

1. Trees that are cut down and stripped of their branches for timber are approximately cylindrical. A timber company specializes in a certain type of tree that has a typical diameter of 50 cm and a typical height of about 10 meters. The density of the wood is 380 kilograms per cubic meter, and the wood can be sold by mass at a rate of $4.75 per kilogram. Determine and state the minimum number of whole trees that must be sold to raise at least $50,000.

$d = 50cm$
$r = 25cm$
or
$r = .25m$

K H D U D C m
m cm
50
.25m 25 cm

① Find volume:
$V = \pi r^2 h$
$V = \pi(.25)^2(10)$
$V = .625\pi$

② Find mass:
$m = D \cdot V$
$m = 380 \cdot .625\pi$
$m = 746.1282 Kg$ for 1 tree

③ How much $ for 1 tree
$746.1282 \times 4.75 =$
$3544.11/tree

④ How many trees:
$\frac{50{,}000}{3544.11} = 14.1079$

$\boxed{\approx 15 \text{ trees}}$

2. A contractor needs to purchase 500 bricks. The dimensions of each brick are 5.1 cm by 10.2 cm by 20.3 cm, and the density of each brick is 1920 kg/m^3. The maximum capacity of the contractor's trailer is 900 kg. Can the trailer hold the weight of 500 bricks? Justify your answer.

5.1cm = .051m KHDUDCm

10.2cm = .102m m cm

20.3cm = .203m

① Find volume

$V = l \cdot w \cdot h$

$V = .051 \cdot .102 \cdot .203$

$V = .00105606$

for 1 brick

x 500

$V = .528003 m^3$

for 500 bricks

② Find mass

$m = D \cdot V$

$m = 1920 \cdot .528003$

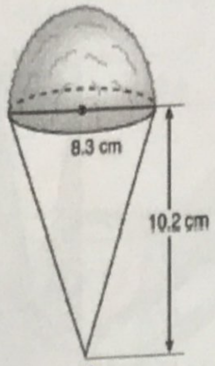
$m = 1013.7657$ KG

The trailer can NOT hold the 500 bricks b/c it exceeds the 900 kg limit.

3. A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.

D = 8.3cm

r = 4.15cm



K H D U D C M

kg g

0.697g

.000697kg = D

.000697

The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

① Find volume

cone + Hemisphere

$\frac{1}{3}\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$\frac{1}{3}\pi(4.15)^2(10.2) + \frac{1}{2}\left(\frac{4}{3}\pi(4.15)^3\right)$

V = 333.6541 for 1 cone

× 50

166682.7078 for 50 cones

② Find mass:

m = D · V

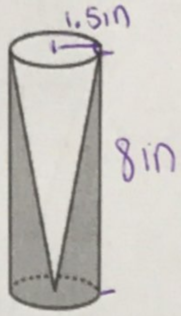
m = .000697 · 166682.7078

m = 11.6278kg for 50 cones

③ Find cost

11.6278kg × 3.83 = $\boxed{\$44.53}$

4. Walter wants to make 100 candles in the shape of a cone for his new candle business. The mold shown below will be used to make the candles. Each mold will have a height of 8 inches and a diameter of 3 inches. To the *nearest cubic inch*, what will be the total volume of 100 candles?



Walter goes to the hobby store to buy the wax for his candles. The wax costs $0.10 per ounce. If the weight of the wax is 0.52 ounce per cubic inch, how much will it cost Walter to buy the wax for 100 candles? If Walter spent a total of $37.83 for the molds and charges $1.95 for each candle, what is Walter's profit after selling 100 candles?

Density

① Find volume:

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(1.5)^2(8)$

$V = 6\pi$ for 1 candle

x100

1884.9555 for 100 candles

$\boxed{V = 1885 \text{ in}^3}$

② Find mass:

$m = D \cdot V$

$m = .52 \cdot 1885$

$\boxed{m = 980.2 \text{ oz}}$

③ Find cost:

980.2 x .10 = $\boxed{\$98.02 \text{ for wax}}$

④ Find profit

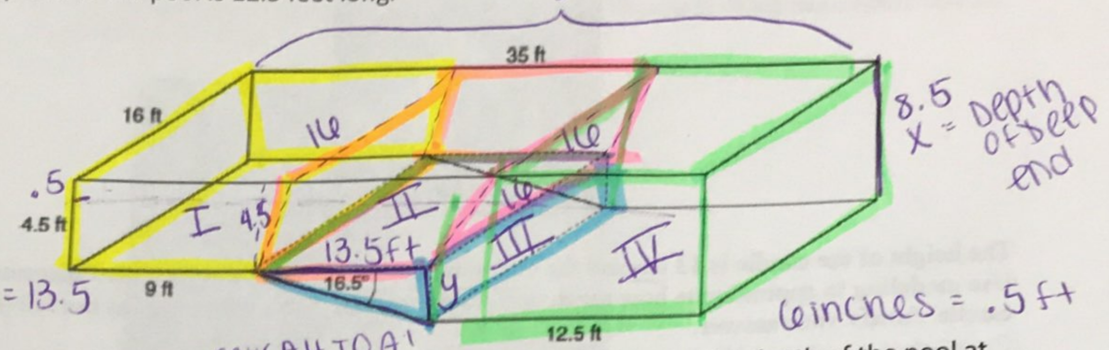
How much he made − How much he spent

100 x 1.95   98.02 + 37.83

$195 − $135.85

$\boxed{\$59.15 \text{ profit}}$

5. A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



$35 - 9 - 12.5 = 13.5$

6 inches = .5 ft

SOHCAHTOA!

If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot?* Find the volume of the inside of the pool to the *nearest cubic foot*. A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top? $[1\ ft^3 = 7.48\ gallons]$

① Find Depth: *$4.5 + y = \text{Depth}$ *

$\frac{\tan 16.5}{1} = \frac{y}{13.5}$

$y = 13.5 \tan 16.5$

$y = 3.9988$

$+4.5$

8.4988

$\boxed{8.5 \text{ ft}}$

② Find volume

V_I: $16 \times 4.5 \times 9 = 648 ft^3$

V_{II}: $4.5 \times 13.5 \times \cancel{9}\,16 = 972 ft^3$

*Triangular prism!

V_{III}: $\frac{1}{2}(13.5)(4)(16) = 432 ft^3$

V_{IV}: $12.5 \times 16 \times 8.5 = 1700 ft^3$

Total volume = ~~3507~~ $\boxed{3752 ft^3}$

③ $3752 - \underbrace{(35)(16)(\tfrac{1}{2})}_{\text{6in from top}} = \underbrace{3472 ft^3}_{\text{filled w/ water}}$

$3472 \times 7.48 = 25970.56$ gallons

④ $\frac{25970.56}{10.5\ \text{gal/min}} = 2473.3866$ minutes

$\frac{2473.3866}{60} = \boxed{41 \text{ hours}}$

6.

A candle maker uses a mold to make candles like the one shown below.

The height of the candle is 13 cm and the circumference of the candle at its widest measure is 31.416 cm. Use modeling to approximate how much wax, to the *nearest cubic centimeter*, is needed to make this candle. Justify your answer.

① Find radius

$C = 2\pi r$

$\frac{31.416}{2\pi} = \frac{2\pi r}{2\pi}$

$r = 5.000$ cm

② Find volume

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(5.000)^2 \cdot 13$

$\boxed{V = 340 \text{ cm}^3}$

7.

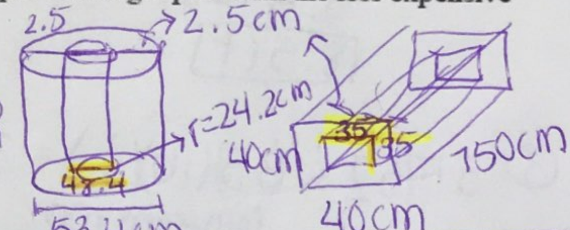
New streetlights will be installed along a section of the highway. The posts for the streetlights will be 7.5 m tall and made of aluminum. The city can choose to buy the posts shaped like cylinders or the posts shaped like rectangular prisms. The cylindrical posts have a hollow core, with aluminum 2.5 cm thick, and an outer diameter of 53.4 cm. The rectangular-prism posts have a hollow core, with aluminum 2.5 cm thick, and a square base that measures 40 cm on each side. The density of aluminum is 2.7 g/cm3, and the cost of aluminum is \$0.38 per kilogram. If all posts must be the same shape, which post design will cost the town less? How much money will be saved per streetlight post with the less expensive design?

① conversions KHDUDCM

7.5 m = 750 cm

m cm

750



$\boxed{r = 26.7}$

② Volume cylinder

$V_{solid} - V_{hollow}$

$\pi(26.7)^2(750) - \pi(24.2)^2(750)$

$299825.7489 \text{ cm}^3 \cdot \frac{2.7 g}{\text{cm}^3}$ (convert to kg!)

809529.522 g (convert to kg)

809.5295 kg

x .38

$\boxed{\$307.621}$

③ volume rect. prisim

$V_{solid} - V_{hollow}$

$(40)(40)(750) - (35)(35)(750)$

$281250 \text{ cm}^3 \cdot \frac{2.7 g}{\text{cm}^3}$

759375 g (convert to kg)

759.375 kg

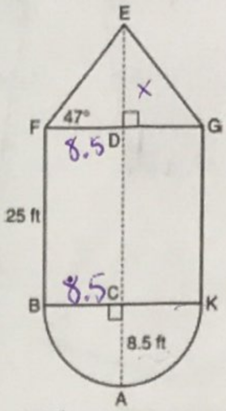
x .38

$\boxed{\$288.56}$ \$19.06 saved

8.

The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.

Source: http://en.wikipedia.org



If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, **determine and state**, to the *nearest cubic foot*, the volume of the water tower. The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

① solve for ED

$\frac{\tan 47}{1} = \frac{x}{8.5}$

$x = 8.5\tan 47$

$x = 9.1151$

② volume of cone

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (8.5)^2 (9.1151)$

$V = \cancel{2068.9537}$ 689.65125 ft³

③ volume of cylinder

$V = \pi r^2 h$

$V = \pi (8.5)^2 (25)$

$V = 5674.5017$ ft³

④ volume of hemisphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{1}{2}\left(\frac{4}{3}\pi (8.5)^3\right)$

$V = 1286.2203$ ft³

④ Total volume

689.65125 + 5674.5017 + 1286.2203

7650.3733

$\boxed{7650\text{ ft}^3}$

⑤ convert to lbs

$7650\ \cancel{\text{ft}^3} \cdot \frac{62.4\text{ lbs}}{1\ \cancel{\text{ft}^3}}$

477360 lbs

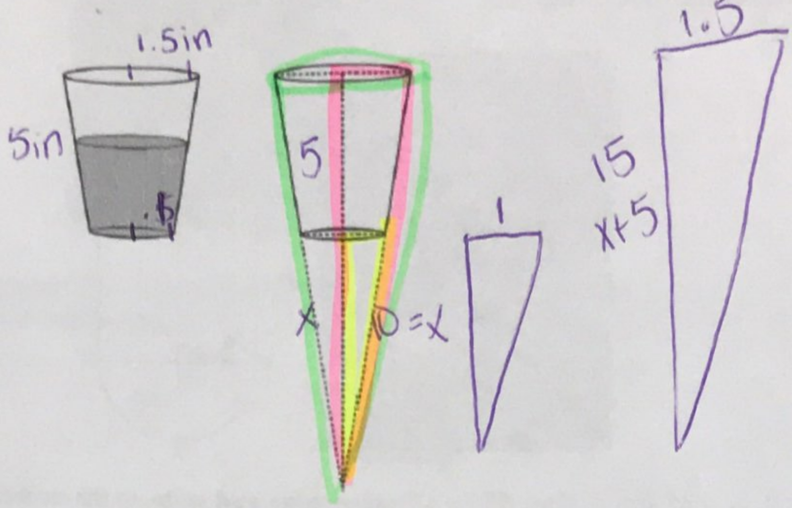
⑥ 85% of volume

477360 × .85 = 405756 lbs

$\boxed{\text{NO } \cancel{\text{yes}} \text{ b/c 405,756 lbs exceeds the 400,000 lb limit}}$

9.

A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.



The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches. The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why. Determine and state, in inches, the height of the larger cone. Determine and state, to the *nearest tenth of a cubic inch*, the volume of the water glass.

① They must be ∥ b/c if 2 Δ's are ≅, the corresponding sides are in proportion

② $\frac{x+5}{x} = \frac{1.5}{1}$

$x+5 = 1.5x$

$-x \quad\quad -x$

$5 = .5x$

$\frac{5}{.5} = \frac{.5x}{.5}$

$\boxed{x=10 \rightarrow \text{height of smaller cone}}$

$\boxed{x=15 \rightarrow \text{height of larger cone}}$

③ volume of water glass

$V_{\text{Big cone}} - V_{\text{small cone}}$

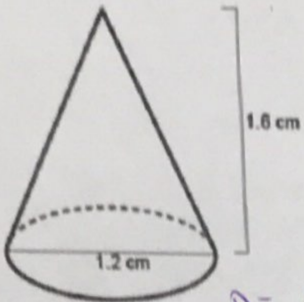
$\frac{1}{3}\pi(1.5)^2(15) - \frac{1}{3}\pi(1)^2(10)$

24.8709

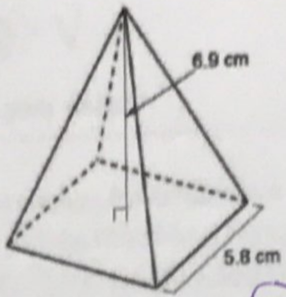
$\boxed{24.9 \text{ in}^3}$

10.

Luke is comparing the value of two small metal statues, one made of solid silver and one made of solid gold. As shown below, the silver statue is in the shape of a regular pyramid with a square base, which has a side length of 5.8 cm and a height of 6.9 cm, and the gold statue is in the shape of a right circular cone with a diameter of 1.2 cm and a height of 1.6 cm. The density of silver is 10.49 g/cm3 and the density of gold is 19.32 g/cm3.!



Gold Statue



Silver Statue

D = 1.2
r = .6

⑦ $\frac{77.372}{.6031} = 128.2904$

$\boxed{128\ \text{statues}}$

mass!

Determine the weight of each statue, to the *nearest gram.*
If silver is valued at \$0.53 per gram and gold is valued at \$38.17 per gram, use the weights Found above to determine which statue has a higher value *and* by how much, to the *nearest cent.*

If both statues were melted down, about how many gold statues, to the *nearest whole number*, would need to be melted in order to have the same volume as a single silver statue?

① Volume of Gold statue

$V = \frac{1}{3}\pi(.6)^2(1.6)$

$V = .6031\ cm^3$

② Volume of Silver statue

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}s^2h$

$V = \frac{1}{3}(5.8)^2(6.9)$

$V = 77.372\ cm^3$

③ Find mass

$m = D \cdot V$

$m = 19.32 \times .603$

$m = 11.6535$

$m = 12\ \text{grams}$

④ Find mass

$m = D \cdot V$

$m = 10.49 \cdot 77.372$

$m = 811.63228$

$m = 812\ \text{grams}$

⑤ Find price

$12g \times 38.17 = \$458.04$

⑥ Find price

$812g \times .53 = \$430.36$

⑥ Find higher value: $458.04 - 430.36 = \$27.68$

$\boxed{\text{Gold has a higher value by } \$27.68}$