Name: Key   Date: 1/9/18

CC ALGEBRA 2   TROICI

LESSON #6: DERIVING, SIMPLIFYING & PROVING TRIG IDENTITIES

Do Now:

a. If $\sec\theta = \frac{1}{\cos\theta}$, then what do you think $\sec^2\theta = ?$ $\frac{1}{\cos^2\theta}$	b. If $\csc\theta = \frac{1}{\sin\theta}$, then what do you think $\csc^2\theta = ?$ $\frac{1}{\sin^2\theta}$
c. If $\cot\theta = \frac{1}{\tan\theta}$, then what do you think $\cot^2\theta = ?$ $\frac{1}{\tan^2\theta}$ or $\frac{\cos^2\theta}{\sin^2\theta}$	d. If $\tan\theta = \frac{\sin\theta}{\cos\theta}$, then what do you think $\tan^2\theta = ?$ $\frac{\sin^2\theta}{\cos^2\theta}$

THE MAIN TRIG PYTHAGOREAN IDENTITY:

$$\cos^2\theta + \sin^2\theta = 1$$

Deriving the 4 other Pythagorean identities:

a) Solve for $\cos^2\theta$ $\cos^2\theta + \sin^2\theta = 1$ $-\sin^2\theta \quad -\sin^2\theta$ $\boxed{\cos^2\theta = 1 - \sin^2\theta}$	b) Solve for $\sin^2\theta$ $\cos^2\theta + \sin^2\theta = 1$ $-\cos^2\theta \quad -\cos^2\theta$ $\boxed{\sin^2\theta = 1 - \cos^2\theta}$
c) divide each term by $\cos^2\theta$ and simplify $\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ $\boxed{1 + \tan^2\theta = \sec^2\theta}$	d) divide each term by $\sin^2\theta$ and simplify $\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$ $\boxed{\cot^2\theta + 1 = \csc^2\theta}$

TRIGONOMETRIC IDENTITIES

QUOTIENT IDENTITIES	RECIPROCAL IDENTITIES	PYTHAGOREAN IDENTITIES
[illegible]	$\csc\theta = \frac{1}{\sin\theta}$	$\cos^2\theta + \sin^2\theta = 1$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cos^2\theta = 1 - \sin^2\theta$
$\cot\theta = \frac{\cos\theta}{\sin\theta}$	$\cot\theta = \frac{1}{\tan\theta}$	$\sin^2\theta = 1 - \cos^2\theta$
		$\sec^2\theta = 1 + \tan^2\theta$
		$\csc^2\theta = 1 + \cot^2\theta$

PRACTICE WITH IDENTITIES

1) Rewrite the expression $\cos\theta - \cos\theta\sin^2\theta$ in terms of a single trigonometric function.

$\cos\theta - \cos\theta(1-\cos^2\theta)$

$\cos\theta - \cos\theta + \cos^3\theta$

$\boxed{\cos^3\theta}$

2) Rewrite each of the following expressions as a single term.

a) $\dfrac{\cos^2\theta+\sin^2\theta}{\tan^2\theta} = \dfrac{1}{\tan^2\theta} = \boxed{\cot^2\theta}$

b) $\left(\dfrac{1}{1-\sin(x)}\right)\left(\dfrac{1}{1+\sin(x)}\right) = \dfrac{1}{\cos^2 x} = \boxed{\sec^2 x}$

$(1-\sin x)(1+\sin x)$

$1+\sin x - \sin x - \sin^2 x$

$1-\sin^2 x = \cos^2 x$

☆ CONVERT TO SIN θ OR COS θ ☆

3) Prove that the equation is an identity:

$\sec^2\theta = 1+\tan^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

a) $\cos^2\theta(\sec^2\theta-1)=\sin^2\theta$

$\cos^2\theta(\tan^2\theta)=\sin^2\theta$

$\cos^2\theta\left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)=\sin^2\theta$

$\sin^2\theta=\sin^2\theta$ ✓

b) $\tan(\theta)\sin(\theta)+\cos(\theta)=\sec(\theta)$

$\left(\dfrac{\sin\theta}{\cos\theta}\right)\left(\dfrac{\sin\theta}{1}\right)+\left(\dfrac{\cos\theta}{1}\right)=\dfrac{1}{\cos\theta}$

$\cos\theta\left(\dfrac{\sin^2\theta}{\cos\theta}+\dfrac{\cos\theta}{1}=\dfrac{1}{\cos\theta}\right)$

$\sin^2\theta + \cos^2\theta = 1$ ✓

PARTNER PRACTICE:

Rewrite each of the following expressions as a single term.

4) $(\csc^2\theta)(\sin^2\theta)-\sin^2\theta$

$\left(\frac{1}{\sin^2\theta}\right)\left(\frac{\sin^2\theta}{1}\right)-\sin^2\theta$

$1-\sin^2\theta$

$\boxed{\cos^2\theta}$

5) $\dfrac{\cos^2\theta}{1-\cos^2\theta}$

$\frac{\cos^2\theta}{\sin^2\theta}=\boxed{\cot^2\theta}$

6) $1-\dfrac{\tan^2\theta}{\sec^2\theta}$

$1-\dfrac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$

$1-\sin^2\theta$

$\boxed{\cos^2\theta}$

7) $\dfrac{\cot^2\theta}{1-\sin^2\theta}$ is equivalent to . . .

a) $\cos^2\theta$ c) $\csc^2\theta$

b) $\tan^2\theta$ d) $\sin^2\theta-1$

$\dfrac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{\cos^2\theta}{1}}$

$\frac{\cos^2\theta}{\sin^2\theta}\cdot\frac{1}{\cos^2\theta}=\frac{1}{\sin^2\theta}$

$=\csc^2\theta$

Prove the identity:

8) $(1+\cos\theta)(1-\cos\theta)=\sin^2\theta$

$1-\cos^2\theta=\sin^2\theta$ ✓

$\sin^2\theta=\sin^2\theta$

9) $\sec^2\theta(1-\cos^2\theta)=\tan^2\theta$

$\frac{1}{\cos^2\theta}\left(\frac{\sin^2\theta}{1}\right)=\frac{\sin^2\theta}{\cos^2\theta}$

$\frac{\sin^2\theta}{\cos^2\theta}=\frac{\sin^2\theta}{\cos^2\theta}$ ✓

10) $\tan x=\dfrac{\sec x}{\csc x}$

$\frac{\sin x}{\cos x}=\dfrac{\frac{1}{\cos x}}{\frac{1}{\sin x}}$

$\frac{1}{\cos x}\cdot\frac{\sin x}{1}$

$\frac{\sin x}{\cos x}=\frac{\sin x}{\cos x}$ ✓

11) $\cot x+\tan x=(\sec x)(\csc x)$

$\cos x\sin x\left[\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}=\left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)\right]$

$\cos^2x+\sin^2x=1$

$1=1$ ✓

Name: ______________________________________ Date: ____________

CC ALGEBRA 2 TROICI

LAB #17

1. Rewrite the expression $(1-\cos^2\theta)(\csc\theta)$ in terms of a single trigonometric function

2. Rewrite $\sin x - \sin x \bullet \cos^2 x$ as an expression containing a single term.

3. If tan *A* = .8, and A is in Quadrant III, find the sec A using the identity. $\cos^2\theta + \sin^2\theta = 1$
(HINT: Change .8 to a fraction)

4. Prove: $\dfrac{1-\sin^2\theta}{\sin\theta} \bullet \sec\theta = \cot\theta$

5. Prove: $\sin\theta(\csc\theta - \sin\theta) = \cos^2\theta$