

LESSON #5: THE PYTHAGOREAN IDENTITY

Do Now:

- a. If $\cos \theta = -\frac{5}{13}$ and $\sin \theta > 0$, find $\sin \theta$. (Hint: Determine what Quadrant θ is in, and draw your "bowtie" triangle!)

$\cos \theta (-), \sin \theta (+) \rightarrow \text{QII}$
 $(-5)^2 + x^2 = (13)^2$
 $25 + x^2 = 169$
 $\sqrt{x^2} = \sqrt{144}$
 $x = 12$

$\frac{5}{13} \frac{A}{C}$

$\boxed{\sin \theta = \frac{12}{13}}$

- b. If $x^2 + y^2 = 25$ and $x = 4$, what does $y = ?$

$$(4)^2 + y^2 = 25$$

$$16 + y^2 = 25$$

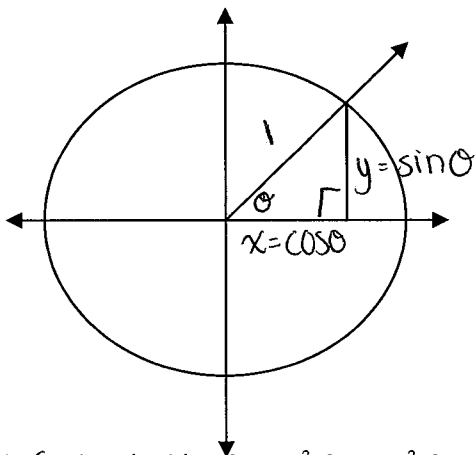
$$\sqrt{y^2} = \sqrt{9}$$

$$\boxed{y = 3}$$

Given the unit circle we can derive the MAIN TRIG IDENTITY!

$$a^2 + b^2 = c^2$$

$$\boxed{\cos^2 \theta + \sin^2 \theta = 1}$$



- 1) If $\cos \theta = -\frac{5}{13}$ and $\sin \theta > 0$, find $\sin \theta$ using the identity $\sin^2 \theta + \cos^2 \theta = 1$. (Do Now question)

still in QII $\rightarrow \sin \theta (+)$

$$\left(-\frac{5}{13}\right)^2 + \sin^2 \theta = 1$$

$$\frac{25}{169} + \sin^2 \theta = \frac{169}{169}$$

$$-\frac{25}{169}$$

$$-\frac{25}{169}$$

$$\sqrt{\sin^2 \theta} = \sqrt{\frac{144}{169}}$$

$$\boxed{\sin \theta = \frac{12}{13}}$$

2) If $\sin \theta = \frac{\sqrt{2}}{5}$, and $\frac{\pi}{2} < \theta < \pi$, find $\sec \theta$ using the identity $\sin^2 \theta + \cos^2 \theta = 1$.

↳ convert to degrees

$$\frac{\pi}{2} \cdot \frac{180}{\pi} = 90^\circ$$

$$90^\circ < \theta < 180^\circ$$

$$\frac{\pi}{1} \cdot \frac{180}{\pi} = 180^\circ$$

QII $\rightarrow \sin(+)$, $\cos(-)$, $\sec(-)$

$$\left(\frac{\sqrt{2}}{5}\right)^2 + \cos^2 \theta = 1$$

$$\frac{2}{25} + \cos^2 \theta = 1$$

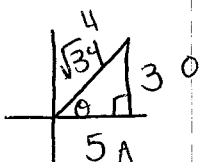
$$\sqrt{\cos^2 \theta} = \sqrt{\frac{23}{25}}$$

$$\cos \theta = \frac{\sqrt{23}}{5}$$

$$\boxed{\sec \theta = -\frac{5}{\sqrt{23}}}$$

3) If $\tan \theta = 0.6$ and θ is a positive acute angle, find $\cos \theta$ using the identity $\sin^2 \theta + \cos^2 \theta = 1$.

$$\tan \theta = \frac{6}{10} = \frac{3}{5} \text{ QI!}$$



$$3^2 + 5^2 = x^2$$

$$9 + 25 = x^2$$

$$34 = x^2$$

$$x = \sqrt{34}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\left(\frac{3}{\sqrt{34}}\right)^2 + \cos^2 \theta = 1$$

$$\frac{9}{34} + \cos^2 \theta = \frac{34}{34}$$

$$\sqrt{\cos^2 \theta} = \frac{\sqrt{25}}{\sqrt{34}}$$

$$\boxed{\cos \theta = \frac{5}{\sqrt{34}}}$$

4) If $\csc \theta = -\frac{9}{4}$, and $\cos \theta < 0$, find the value of $\sec \theta$ using the identity $\sin^2 \theta + \cos^2 \theta = 1$.

$$\sin \theta = -\frac{4}{9}$$

S/A
T/C

↳ need $\cos \theta$ going to be negative!

$$\left(-\frac{4}{9}\right)^2 + \cos^2 \theta = 1$$

$$\frac{16}{81} + \cos^2 \theta = \frac{81}{81}$$

$$\sqrt{\cos^2 \theta} = \frac{\sqrt{65}}{\sqrt{81}}$$

$$\cos \theta = -\frac{\sqrt{65}}{9}$$

$$\boxed{\sec \theta = -\frac{9}{\sqrt{65}}}$$