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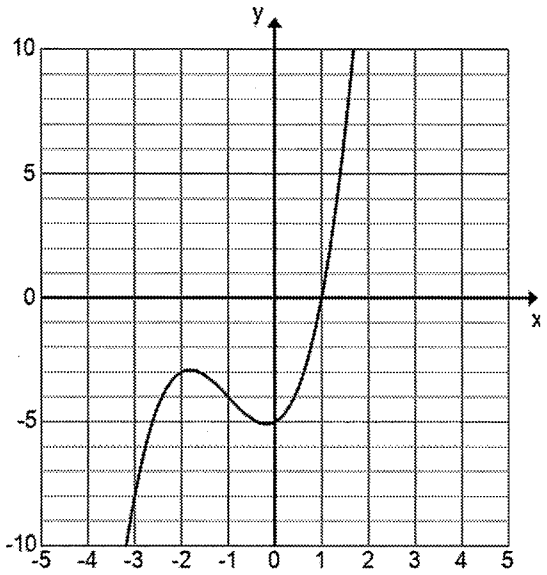
CC ALGEBRA 2

TROICI

LESSON #9: SOLVING HIGHER-DEGREE POLYNOMIAL EQUATIONS WITH COMPLEX ROOTS (DAY 2)

1) Consider the polynomial equation $p(x) = x^3 + 3x^2 + x - 5$.



a) How many real solutions does this equation have? 3

b) How many complex solutions does this equation have? 2 (only hits x-axis 1x)

c) What is the real solution to this equation? 1

d) What linear factor does this solution come from? (x-1)

e) How can we use that linear factor to solve for the other solutions? Divide (long division)

f) Solve for the complex solutions.

① Divide

$$x-1 \overline{)\, x^3+3x^2+x-5}$$

$$x^2+4x+5$$

$$-x^3+x^2$$

$$4x^2+x$$

$$-4x^2+4x$$

$$5x-5$$

$$-5x+5$$

$$0$$

② Factor

$$x^2+4x+5=0$$

$$-5 \quad -5$$

$$x^2+4x+\boxed{4} = -5+\boxed{4}$$

$$\left(\frac{4}{2}\right)^2$$

$$\sqrt{(x+2)^2} = \sqrt{-1}$$

$$x+2 = \pm i$$

$$\boxed{x = -2 \pm i}$$

2) Find the solutions of $x^3 + 4x^2 + 8x + 8 = 0$ if $(x + 2)$ is a factor.

① Divide

$$x+2 \overline{)\, x^3+4x^2+8x+8}$$

$$x^2+2x+4$$

$$-x^3 \mp 2x^2$$

$$2x^2+8x$$

$$-2x^2 \mp 4x$$

$$4x+8$$

$$-4x \mp 8$$

$$0$$

② Factor

$$x^2+2x+4=0$$

$$-4 \quad -4$$

$$x^2+2x+\boxed{1} = -4+\boxed{1}$$

$$\left(\frac{2}{2}\right)^2$$

$$\sqrt{(x+1)^2} = \sqrt{-3}$$

$$x+1 = \pm i\sqrt{3}$$

$$-1 \quad -1$$

$$\boxed{x = -1 \pm i\sqrt{3}}$$

$$\{-2, -1 \pm i\sqrt{3}\}$$

Practice

3) Find the solutions of $5x^3 + 9x^2 - 13x + 15 = 0$ if -3 is a root. (Hint: What is the factor when $x = -3$ is a root?)

$(x+3)$

$$x+3 \overline{)\, 5x^3+9x^2-13x+15}$$

$$5x^2-6x+5$$

$$-5x^3 \mp 15x^2$$

$$-6x^2-13x$$

$$+6x^2+18x$$

$$5x+15$$

$$-5x \mp 15$$

$$0$$

$$5x^2-6x+5=0$$

$a = 5$

$b = -6$

$c = 5$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2-4(5)(5)}}{2(5)}$$

$$x = \frac{6 \pm \sqrt{-64}}{10}$$

$$x = \frac{6 \pm 8i}{10}$$

$$\boxed{x = \frac{3 \pm 4i}{5}}$$

$$\left\{-3, \frac{3 \pm 4i}{5}\right\}$$