

LESSON #8: SOLVING HIGHER-DEGREE POLYNOMIAL EQUATIONS WITH COMPLEX ROOTS

Do Now:

Given: $2x + 3 = 11$

$$\begin{array}{r} -3 \quad -3 \\ \hline 2x = 8 \\ x = 4 \end{array}$$

a. What is the degree?
1

b. How many solutions are there?
1

c. What is the solution(s)?
 $x = 4$

Given: $x^2 - 9 = 0$

$$\begin{array}{r} +9 \quad +9 \\ \hline x^2 = 9 \\ x = \pm 3 \end{array}$$

a. What is the degree?
2

b. How many solutions are there?
2

c. What is the solution(s)?
 $x = \pm 3$

For #1 and 2, solve for all real and complex solutions of the following polynomial equations. Sketch the graph to verify your results.

1) $x^3 + 4x = 0$

$$x(x^2 + 4) = 0$$

$x = 0$	$x^2 + 4 = 0$
	$x^2 = -4$
	$x = \pm 2i$

EB = down, up

a. What is the solution(s)?

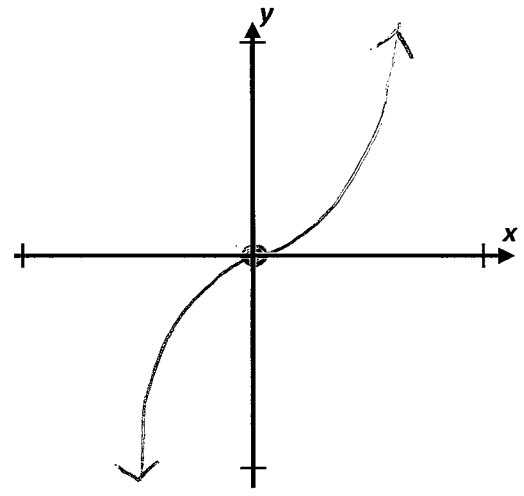
$0, \pm 2i$

b. What is the degree?

3

c. How many solutions are there?

3

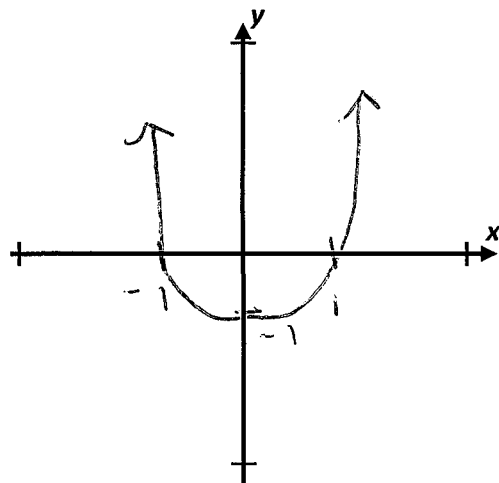


$$2) x^4 - 1 = 0$$

$$(x^2 - 1)(x^2 + 1) = 0$$

$$\begin{array}{l|l} x^2 - 1 = 0 & x^2 + 1 = 0 \\ \sqrt{x^2} = \sqrt{1} & \sqrt{x^2} = \sqrt{-1} \\ x = \pm 1 & x = \pm i \end{array}$$

$$\{ \pm 1, \pm i \}$$



$$3) x^3 + 5x = 0$$

$$x(x^2 + 5) = 0$$

$$\begin{array}{l|l} x = 0 & x^2 + 5 = 0 \\ & \sqrt{x^2} = \sqrt{-5} \\ & x = \pm i\sqrt{5} \end{array}$$

$$\{ 0, \pm i\sqrt{5} \}$$

$$4) x^4 - 5x^2 - 36 = 0$$

$$(x^2 - 9)(x^2 + 4) = 0$$

$$\begin{array}{l|l} \sqrt{x^2} = \sqrt{9} & \sqrt{x^2} = \sqrt{-4} \\ x = \pm 3 & x = \pm 2i \end{array}$$

$$\{ \pm 3, \pm 2i \}$$

$$5) x^3 + 27 = 0 \quad (a+b)(a^2 - ab + b^2) \quad \text{S.A.P.T.}$$

$$(x+3)(x^2 - 3x + 9) = 0$$

$$\begin{array}{l|l} x = -3 & x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)} \end{array}$$

$$x = \frac{3 \pm \sqrt{-27}}{2} = \frac{3 \pm \sqrt{9} \cdot \sqrt{-3}}{2} = \frac{3 \pm 3i\sqrt{3}}{2}$$

$$x = \frac{3 \pm 3i\sqrt{3}}{2}$$

$$\left\{ -3, \frac{3 \pm 3i\sqrt{3}}{2} \right\}$$

Practice:

$$6) x^3 - 5x^2 + 4x - 20 = 0$$

$$x^2(x-5) + 4(x-5)$$

$$(x^2+4)(x-5) = 0$$

$$x^2 = -4 \quad | \quad x = 5$$

$$x = \pm 2i$$

$$\{5, \pm 2i\}$$

$$7) x^3 + 16x = 0$$

$$x(x^2 + 16) = 0$$

$$x = 0 \quad | \quad \sqrt{x^2 = -16}$$

$$x = \pm 4i$$

$$\{0, \pm 4i\}$$

$$8) x^4 - 6x^2 - 7 = 0$$

$$(x^2 - 7)(x^2 + 1) = 0$$

$$\sqrt{x^2 = 7} \quad | \quad \sqrt{x^2 = -1}$$

$$x = \pm\sqrt{7} \quad | \quad x = \pm i$$

$$\{\pm\sqrt{7}, \pm i\}$$

$$9) x^3 + 8 = 0 \quad (a+b)(a^2 + ab + b^2)$$

$$(x+2)(x^2 - 2x + 4)$$

$$-2 \quad | \quad x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm \frac{\sqrt{4}}{\sqrt{3}} \cdot 2i\sqrt{3}}{2}$$

$$x = \frac{2 \pm 2i\sqrt{3}}{2}$$

$$x = 1 \pm i\sqrt{3}$$

$$\{-2, 1 \pm i\sqrt{3}\}$$

