

LESSON #3: OPERATIONS WITH COMPLEX NUMBERS (DAY 2)

Do Now:

Is 2 a solution to $x^2 + 3x - 10 = 0$? Justify your answer.

$$\begin{aligned} (2)^2 + 3(2) - 10 &= 0 \\ 4 + 6 - 10 &= 0 \\ 10 - 10 &= 0 \\ 0 &= 0 \end{aligned}$$

yes, 2 is a solution b/c it satisfies the remainder theorem.

1. Determine if $(-1 + 2i)$ is a solution to $x^2 + 2x + 5 = 0$. Justify your answer.

same idea! plug in!

$$\begin{aligned} (-1 + 2i)^2 + 2(-1 + 2i) + 5 &= 0 \\ (-1 + 2i)(-1 + 2i) + -2 + 4i + 5 &= 0 \\ 1 - 2i - 2i + 4i^2 &\quad \downarrow \quad \downarrow \quad \downarrow \\ 1 - 4i + 4(-1) &\quad \downarrow \quad \downarrow \quad \downarrow \\ 1 - 4i - 4 - 2 + 4i + 5 &= 0 \\ 0 &= 0 \checkmark \end{aligned}$$

yes $(-1 + 2i)$ is a solution!

2. $\frac{i^{10}}{i^4}$

$$\begin{aligned} i^{10-4} &= i^6 \\ &= \boxed{-1} \end{aligned}$$

3. $(3i^2)^4$

$$\begin{aligned} 81i^8 \\ 81(1) &= \boxed{81} \end{aligned}$$

4. $(i^6 - 2)(i^0 + 2)$

$$\begin{aligned} &\text{OR } i^{36} - 4 \\ &1 - 3 = \boxed{-3} \\ (-1 - 2)(-1 + 2) \\ (-3)(1) \\ \boxed{-3} \end{aligned}$$

5. Solve for x: $i^{17} + i^{16} + i^{46} + x = 0$

$$\begin{aligned} i + 1 + 1 + x &= 0 \\ i + x &= 0 \\ -i &\quad -i \\ \boxed{x = -1} \end{aligned}$$

6. Simplify $xi(i + 3i)^2$

$$\begin{aligned} xi(4i)^2 \\ xi(16i^2) \\ xi(16(-1)) \\ xi - 16xi &= \boxed{-16xi} \end{aligned}$$

7. $\frac{\sqrt{-64}}{-\sqrt{16}} = \frac{48i}{-4} = \boxed{-4i}$

8. The expression $(x+i)^2 - (x-i)^2$ is equivalent to:

a) 0

b) -2

c) $-2+4xi$

d) $4xi$

$$\begin{aligned} & (x+i)(x+i) - (x-i)(x-i) \\ & x^2 + xi + xi + i^2 - (x^2 - xi - xi + i^2) \\ & x^2 + 2xi - 1 - (x^2 + 2xi + 1) \\ & \quad \quad \quad 4xi \end{aligned}$$

Practice - Simplify:

9. Simplify: $(3+2i)(3-2i) - 3i(1-2i)$

$$\begin{aligned} & 9 - 4i^2 - (3i + 6i^2) \\ & 9 - 4(-1) - 3i - 6 \\ & 9 + 4 \\ & 13 - 3i - 6 \\ & \boxed{7 - 3i} \end{aligned}$$

11. $\frac{6i^{13}}{2i^5}$ $3i^8$

$$3(1) = \boxed{3}$$

10. Simplify $xi(i-4i)^3$

$$\begin{aligned} & xi(-3i)^3 \\ & xi(-27i^3) \\ & xi(-27-i) \\ & xi(27i) \\ & 27i^2 x \\ & \boxed{-27x} \end{aligned}$$

12. $(-4i^4)^3$

$$\begin{aligned} & -64i^{12} \\ & -64(1) \\ & \boxed{-64} \end{aligned}$$

13. Is $x=3-i$ a solution to $x^2 - 6x = 0$? Justify your answer.

$$\begin{aligned} & (3-i)^2 - 6(3-i) = 0 \\ & (3-i)(3-i) - 18 + 6i = 0 \\ & 9 - 3i - 3i + i^2 - 18 + 6i = 0 \\ & 9 - 6i - 1 - 18 + 6i \\ & \quad \quad \quad -10 \neq 0 \end{aligned}$$

NO, $3-i$ IS NOT a solution!