

LESSON #1: INTRODUCTION TO IMAGINARY NUMBERS

Do Now:

1) Solve each equation for x.

a) $x^2 - 1 = 0$

$$\frac{+1 \quad +1}{\sqrt{x^2} = \sqrt{1}}$$

$$x = \pm 1$$

b) $x^2 + 1 = 0$

$$\frac{-1 \quad -1}{\sqrt{x^2} = \sqrt{-1}}$$

NO SOLUTION

2) Simplify the radical expression.

$$\begin{aligned} & 3\sqrt[3]{72} \\ & 3\sqrt[3]{36} \sqrt{2} \\ & 3 \cdot 6\sqrt{2} \\ & \boxed{18\sqrt{2}} \end{aligned}$$

Solve the following equation again for x:

$x^2 + 1 = 0$ \rightarrow or is there?

$$\frac{-1 \quad -1}{\sqrt{x^2} = \sqrt{-1}}$$

$$x = \pm i$$

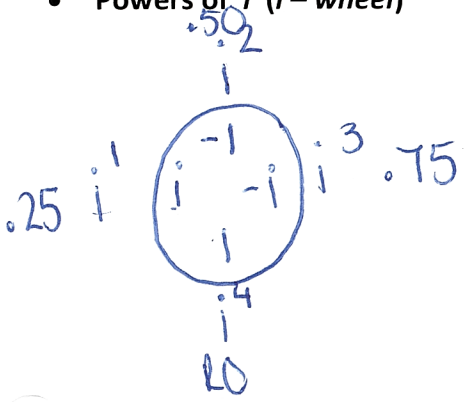
On your calculator, press **MODE** and change it from real to "a + bi", now type in $\sqrt{-1}$.

$\bullet \sqrt{-1} = i$

Must Memorize

$$i^2 = -1$$

• Powers of i (i-wheel)



$i^1 = i$
 $i^2 = i \cdot i = -1$
 $i^3 = i \cdot i \cdot i = -i$
 $i^4 = i \cdot i \cdot i \cdot i = 1$
 $i^5 = i \cdot i \cdot i \cdot i \cdot i = i$
 $i^6 = i \cdot i \cdot i \cdot i \cdot i \cdot i = -1$
 $i^7 = i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = -i$
 $i^8 = i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = 1$
 $i^{15} = i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = -i$
 $\frac{15}{4} = 3 \cdot 75 = i^{\textcircled{3}} = -i$

Divide exponent by 4
 use remainder to determine power of i

What does i^{125} equal?

$i \text{ part } (i^{125}) = i^{\textcircled{1}} \text{ OR } \frac{125}{4} = 31 \cdot 25 = i^{\textcircled{1}} = i$

Calculate the following powers of i .

1) $i^{247} = -i$

2) $i^{254} = -1$

3) $i^{256} = 1$

• Operations with "i" #'s w/ #'s, i's w/ i's (treat like x)

4) $(3 + 4i) + (-4 + 2i)$

$$\boxed{-1 + 6i}$$

5) $(3 + 2i) - (-1 + i)$

$$\boxed{4 + i}$$

No Calculator (SAT and regents) DOUBLE DISTRIBUTION!

6) $(3 + 4i)(5 - 2i)$
 $15 - 6i + 20i - 8i^2 \rightarrow -1$
 $15 + 14i - 8(-1)$
 $15 + 14i + 8 = \boxed{23 + 14i}$

7) $(4 + 3i)(4 - 3i)$ CONJUGATE!
 $16 - 12i + 12i - 9i^2 \rightarrow -1$
 $16 - 9(-1)$
 $16 + 9 = \boxed{25}$

8) $(1 - 6i)^2 = (1 - 6i)(1 - 6i)$
 $1 - 6i - 6i + 36i^2 \rightarrow -1$
 $1 - 12i + 36(-1)$
 $1 - 12i - 36 = \boxed{-35 - 12i}$

9) $(6 - 2i)(6 + 2i)$ CONJUGATE
 $36 - 4i^2 \rightarrow -1$
 $36 - 4(-1)$
 $36 + 4 = \boxed{40}$

Practice: Calculate the powers of i .

10) $i^{23} = -i$

11) $i^{102} = -1$

12) $i^{201} = i$

13) $i^{448} = 1$

Simplify: (Show all substitutions for terms with "i").

14) $(3 + 4i) + (-4 + 2i)$

$$\boxed{-1 + 6i}$$

15) $(3 + 2i) - (-1 + i)$

$$\boxed{4 - i}$$

16) $(1 + 6i)(3 - 4i)$
 $3 - 4i + 18i - 24i^2$
 $3 + 14i - 24(-1)$
 $3 + 14i + 24$

$$\boxed{27 + 14i}$$

17) $(7 + 2i)(7 - 2i)$ CONJUGATE
 $49 - 4i^2$
 $49 + 4$

$$\boxed{53}$$