

UNIT 4- REVIEW FOR TEST- SYSTEMS OF EQUATIONS + PARABOLAS

****YOUR TEST IS WEDNESDAY 11/29/17!****

Solve each system of equations and check on the calculator using 2nd matrix!

1) $x - z = 5$ (6pt)
 $2x + y = 7$
 $y + 3z = 12$

$$\begin{array}{r} 3(x - z = 5) \\ y + 3z = 12 \end{array} \quad \begin{array}{r} 3x - 3z = 15 \\ + y + 3z = 12 \\ \hline 3x + y = 27 \end{array}$$

$$\begin{array}{r} 3x + y = 27 \\ - (2x + y = 7) \end{array} \quad \begin{array}{r} 3x + y = 27 \\ + -2x - y = -7 \\ \hline x = 20 \end{array}$$

$$\begin{array}{l} 2x + y = 7 \\ 2(20) + y = 7 \\ y = -33 \end{array}$$

$$y + 3z = 12$$

$$-33 + 3z = 12$$

$$3z = 45$$

$$z = 15$$

CHECK:

$$(20) - (15) = 5 \checkmark$$

$$2(20) + (-33) = 7 \checkmark$$

$$(-33) + 3(15) = 12 \checkmark$$

$$\begin{array}{l} x = 20 \\ y = -33 \\ z = 15 \end{array}$$

2) Find the equation of a parabola in $y = ax^2 + bx + c$ form that passes through the points (1, 3), (2, 2) and (3, -3). Check on your calculator by typing your final equation into $y =$ and look at your table for the 3 given points! (6pt)

For point (1, 3): $3 = a(1)^2 + b(1) + c$
 (2, 2): $2 = a(2)^2 + b(2) + c$
 (3, -3): $-3 = a(3)^2 + b(3) + c$

$$\begin{cases} a + b + c = 3 \\ 4a + 2b + c = 2 \\ 9a + 3b + c = -3 \end{cases}$$

$$\begin{array}{r} a + b + c = 3 \\ - (4a + 2b + c = 2) \\ \hline -3a - b = 1 \end{array} \quad \begin{array}{r} a + b + c = 3 \\ - (9a + 3b + c = -3) \\ \hline -8a - 2b = 6 \end{array}$$

$$\begin{array}{r} 4a + 2b + c = 2 \\ - (9a + 3b + c = -3) \\ \hline -5a - b = 5 \end{array}$$

$$\begin{array}{r} -3a - b = 1 \\ - (-5a - b = 5) \\ \hline 2a = -4 \\ a = -2 \end{array}$$

$$\begin{array}{l} a + b + c = 3 \\ (-2) + (5) + c = 3 \\ 3 + c = 3 \\ c = 0 \end{array}$$

CHECK:
 $4(-2) + 2(5) + 0 = 2$
 $-8 + 10 = 2$
 $2 = 2 \checkmark$

$$\begin{array}{l} -3(-2) - b = 1 \\ 6 - b = 1 \\ b = 5 \end{array}$$

3) Find the center and radius of the following circles:

a) $x^2 + y^2 = 36$ (2pt)
 center = (0,0)
 radius = 6

b) $(x-2)^2 + (y+1)^2 = 16$ (2pt)
 center = (2,-1)
 radius = 4

c) $x^2 + y^2 + 6x - 8y + 9 = 0$ (4pt)
 $x^2 + \frac{6x}{2} + \frac{9}{2} + y^2 - 8y + \frac{16}{2} = -9 + \frac{9}{2} + \frac{16}{2}$
 $(x+3)^2 + (y-4)^2 = 16$
 center = (-3,4)
 radius = 4

Find the coordinates of the point(s) of intersection for each system of equations algebraically and illustrate graphically.

4) $x^2 + y^2 = 25$: center = (0,0) radius = 5 (4pt)
 $2y - 2x = -2$
 $\quad + 2x \quad + 2x$
 $\frac{2y}{2} = \frac{2x-2}{2}$
 $y = x-1 \rightarrow \text{slope} = 1$
 $y_{int} = -1$

$x^2 + (x-1)^2 = 25$
 $x^2 + (x-1)(x-1) = 25$
 $x^2 + x^2 - x - x + 1 = 25$

$2x^2 - 2x + 1 = 25$
 $\quad -25 \quad -25$

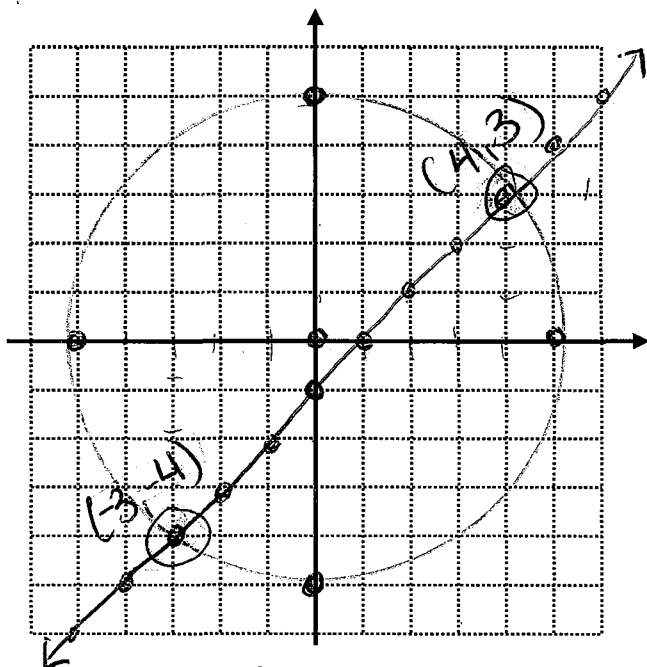
$\frac{2x^2 - 2x - 24}{2} = \frac{0}{2}$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$
 (4) | (-3)

$x = 4$
 $2y - 2(4) = -2$
 $2y - 8 = -2$
 $2y = 6$
 $y = 3$
 (4,3)

$x = -3$
 $2y - 2(-3) = -2$
 $2y + 6 = -2$
 $2y = -8$
 $y = -4$
 (-3,-4)



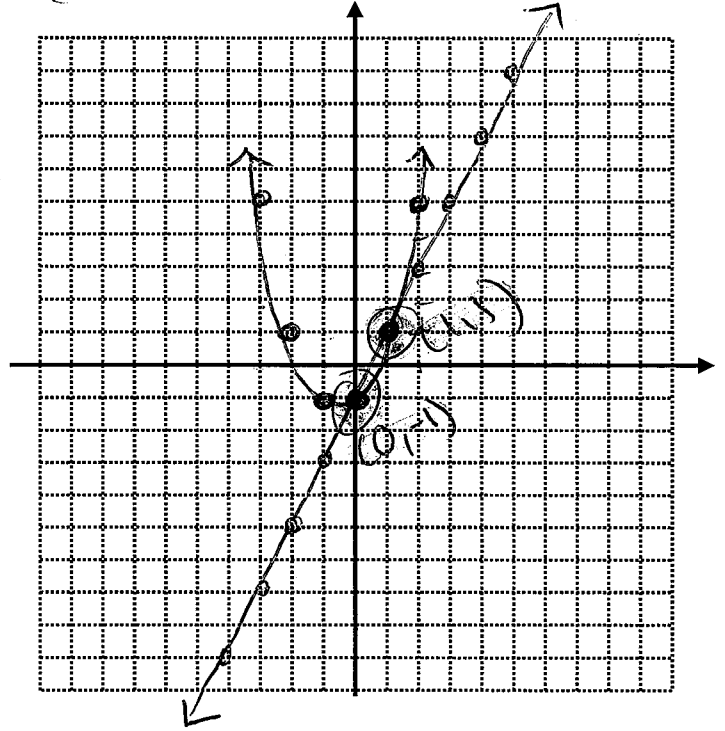
Find the coordinates of the point(s) of intersection for each system of equations *algebraically* and illustrate *graphically*.

5) $y = x^2 + x - 1$; $y = 2x - 1$ → slope = $\frac{2}{1}$ (6pt)
 y int = -1

$$\begin{array}{r} x^2 + x - 1 \\ -2x + 1 \\ \hline x^2 - x = 0 \end{array}$$

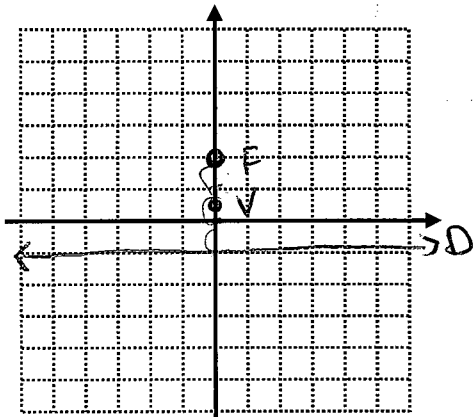
$$x(x-1) = 0$$

$x=0$	$x=1$
$y=2(0)-1$	$y=2(1)-1$
$y=-1$	$y=1$
$(0, -1)$	$(1, 1)$



6) Write the equation of a parabola defined by the given directrix and focus.

focus at $(0, 3)$ and directrix $y = -1$ 4pt



vertex = $(0, 0.5)$
 $p = 3$

$$y - k = \frac{1}{2p} (x - h)^2$$

$$y - 0.5 = \frac{1}{2(3)} (x - 0)^2$$

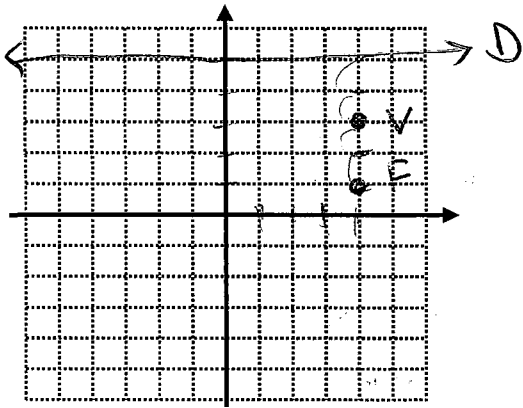
$$y - 0.5 = \frac{1}{6} (x)^2$$

$$y = \frac{1}{6} x^2 + 0.5$$

7. Write the equation of a parabola defined by the given directrix and focus.

focus at (4, 1) and directrix $y = 5$ (4pt)
 vertex = $(4, 3)$
 $p = -4$

* DIRECTRIX IS ABOVE SO P MUST BE (-) ! *



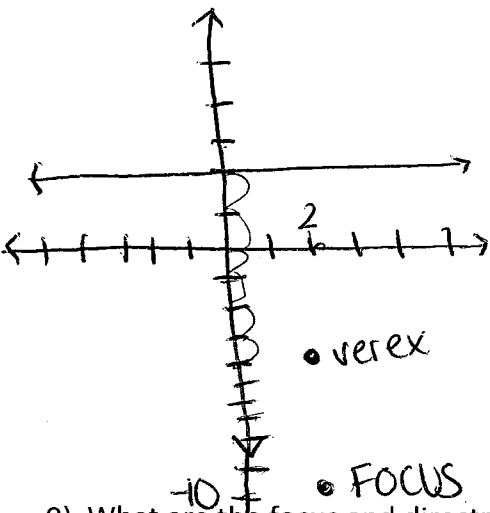
$$y - k = \frac{1}{2p} (x - h)^2$$

$$y - 3 = \frac{1}{2(-4)} (x - 4)^2$$

$$y - 3 = -\frac{1}{8} (x - 4)^2$$

$$y = -\frac{1}{8} (x - 4)^2 + 3$$

8) The directrix of the parabola $\frac{-24(y+4)}{-24} = \frac{(x-2)^2}{-24}$ has the equation $y = 2$. Find the coordinates of the focus. (4pt)



Get y by itself!

$$y + 4 = -\frac{1}{24} (x - 2)^2$$

$$y = -\frac{1}{24} (x - 2)^2 - 4$$

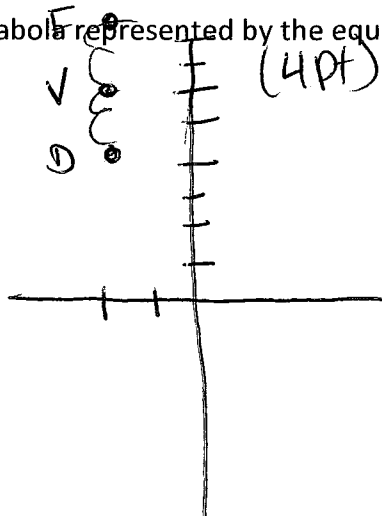
FOCUS (2, -10)

$p = -12$ b/c $2p = 24$
 $p = -12$
 vertex = (2, -4)

9) What are the focus and directrix of the parabola represented by the equation: $\frac{8(y-6)}{8} = \frac{(x+2)^2}{8}$? (4pt)

- (1) Focus: (0, 6)
Directrix: $x = -4$
- (3) Focus: (-2, 4)
Directrix: $y = 8$

- (2) Focus: (-2, 8)
Directrix: $y = 4$
- (4) Focus: (-2, 6)
Directrix: $y = 4$



$$y - k = \frac{1}{8} (x + 2)^2$$

$$y = \frac{1}{8} (x + 2)^2 + 6$$

vertex = (-2, 6)
 $p = 4$

10) Simplify the expression and state any undefined value(s). 4pt

$$\frac{x^3 + 3x^2 - 4x - 12}{x^2 - 4}$$

$$\begin{aligned} & x^2(x+3) - 4(x+3) \\ & \frac{(x^2 - 4)(x+3)}{(x^2 - 4)} \end{aligned}$$

$$\boxed{x+3, x \neq \pm 2}$$

11) Solve for y: $\frac{(y-4)(y+1)}{y-4} + \frac{2}{y+1} = \frac{y-5}{y^2-3y-4}$ 6pt

$$y(y+1) + 2(y-4) = y-5$$

$$y^2 + y + 2y - 8 = y - 5$$

$$\begin{array}{r} y^2 + 3y - 8 = y - 5 \\ -y + 5 \quad -y + 5 \\ \hline \end{array}$$

$$y^2 + 2y - 3 = 0$$

$$\begin{array}{r} (y+3)(y-1) = 0 \\ -3 \quad 1 \end{array}$$

$$\boxed{\{-3, 1\}}$$

12) What is the solution set of the equation $x^2 = (2\sqrt{2x-3})^2$ 4pt

$$\boxed{\{2, 6\}}$$

$$x^2 = 4(2x-3)$$

$$\begin{array}{r} x^2 = 8x - 12 \\ -8x + 12 \end{array}$$

$$x^2 - 8x + 12 = 0$$

$$\begin{array}{r} (x-6)(x-2) = 0 \\ 6 \quad 2 \end{array}$$

check:

$$x = 6$$

$$6 = 2\sqrt{2(6)-3}$$

$$6 = 2\sqrt{9}$$

$$6 = 2(3) \checkmark$$

$$x = 2$$

$$2 = 2\sqrt{2(2)-3}$$

$$2 = 2\sqrt{1}$$

$$2 = 2(1) \checkmark$$

13) Given the polynomial function $P(x) = \frac{x^3 + 4x^2 + x + 6}{x-1}$. (4pt)

a. Divide P by the divisor $(x-1)$ and write the quotient with a rational remainder.

$$\begin{array}{r}
 x^2 + 5x + 6 \\
 x-1 \overline{) x^3 + 4x^2 + x + 6} \\
 \underline{-x^3 + x^2} \\
 5x^2 + x \\
 \underline{-5x^2 + 5x} \\
 6x + 6 \\
 \underline{-6x + 6} \\
 12
 \end{array}$$

$$\boxed{x^2 + 5x + 6 + \frac{12}{x-1}}$$

b. Is $x-1$ a factor of $P(x) = x^3 + 4x^2 + x + 6$? Justify using the remainder theorem (2pt)

$$P(1) = 1^3 + 4(1)^2 + 1 + 6$$

$$P(1) = 12$$

NO, $x-1$ is not a factor b/c the remainder will be 12.

14) What type of relationship is indicated by the following set of ordered pairs? Explain. (2pt)

x	y
0	6
1	0
2	-4
3	0
4	18

1st
 -6
 2nd
 2
 3rd
 6

-4
 8
 6

$+4$
 14
 6

$+18$

THIS IS a CUBIC function b/c the pattern is in the 3rd set of differences

15) Answer the questions below based on the accompanying graph of $f(x)$.

a. Function: Odd/Even/Neither: Explain how you know: 2pt

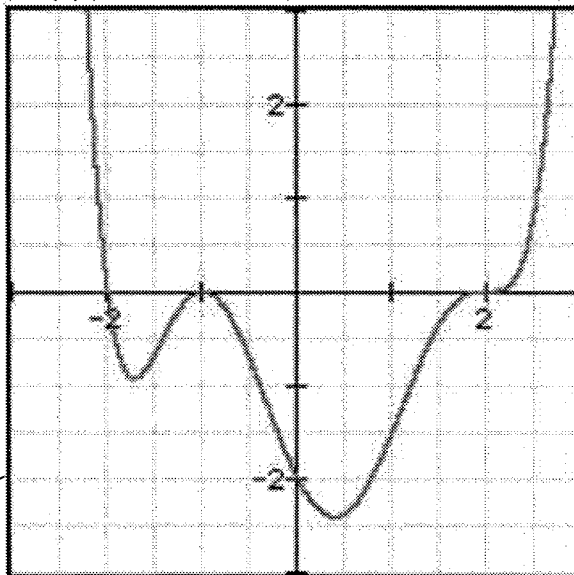
neither, no symmetry

b. Degree: Odd/Even: Explain how you know: 2pt

even, EB = up, up

c. Leading coefficient: Positive/Negative: Explain how you know: 2pt

positive, EB = up, up



d. List the zeros of the function: -2, -1, 2, 3 2pt

16) Solve the system of equation and check on the calculator using 2nd matrix! 6pt

$$\begin{cases} 2x + 4y + z = 1 \\ x - 2y - 3z = 2 \\ x + y - z = -1 \end{cases}$$

$$\begin{array}{r} 2x + 4y + z = 1 \\ x + y - z = -1 \\ \hline 3x + 5y = 0 \end{array}$$

$$\begin{array}{r} 3x + 5y = 0 \\ -2x - 5y = 3 \\ \hline x = 5 \end{array}$$

$$\begin{array}{r} x - 2y - 3z = 2 \\ -3(x + y - z = -1) + 3x - 3y + 3z = 3 \\ \hline -2x - 5y = 5 \end{array}$$

$$\begin{array}{r} 3(5) + 5y = 0 \\ 15 + 5y = 0 \\ -15 \quad -15 \\ \hline 5y = -15 \\ y = -3 \end{array}$$

$$\begin{array}{r} (5) + (-3) - z = -1 \\ 2 - z = -1 \\ -2 \quad -2 \\ \hline -z = -3 \\ z = 3 \end{array}$$

check: $2(5) + 4(-3) + 3 = 1$
 $10 - 12 + 3 = 1$
 $1 = 1 \checkmark$

17) Simplify:

4pt

$$\frac{(x-1)(x-4) \left(\frac{3x+2}{x-1} - \frac{8}{x-4} \right)}{(x-1)(x-4) \left(\frac{12x-2x^2}{x^2-5x+4} \right)}$$

$$\frac{(3x+2)(x-4) - 8(x-1)}{12x - 2x^2}$$

$$3x^2 - 12x + 2x - 8 - 8x + 8$$

$$3x^2 - 18x$$

$$\frac{3x(x-6)}{2x(6-x)} = \boxed{\frac{-3}{2}}$$

18) Using your calculator, find the solutions of $y = -x^3 + 3x^2 + 9x - 11$ and $y = x + 8$? Round your answers to the nearest tenth.

3pt

$$(-2.7, 5.3) \text{ and } (1.9, 9.9) \text{ and } (3.8, 11.8)$$

① ZOOM OUT!

② 2nd → calc → trace

③ ENTER 3X

to