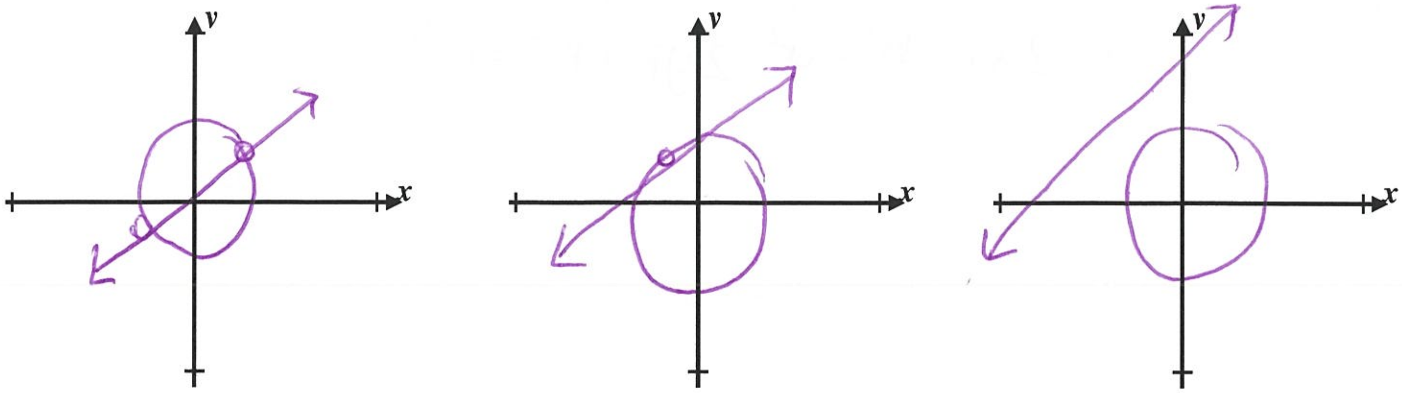
Name: Key Date: 11/16/17

CC ALGEBRA 2 TROICI

LESSON #3: CIRCLE AND LINEAR SYSTEM OF EQUATIONS

Do Now:

a) Suppose you sketch a circle and a line on the same graph. What are the different possibilities that these two figures can intersect?



2 times 1 time (tangent) 0 times

CIRCLES REVIEW:

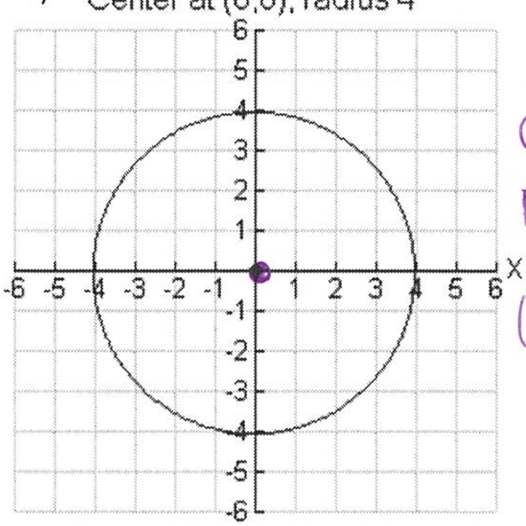
CENTER-RADIUS FORM OF AN EQUATION OF A CIRCLE
$x^2 + y^2 = r^2$
$(x - h)^2 + (y - k)^2 = r^2$

h, k = center

r = radius

Write the equations of the given circles.

a) Center at (0,0), radius 4



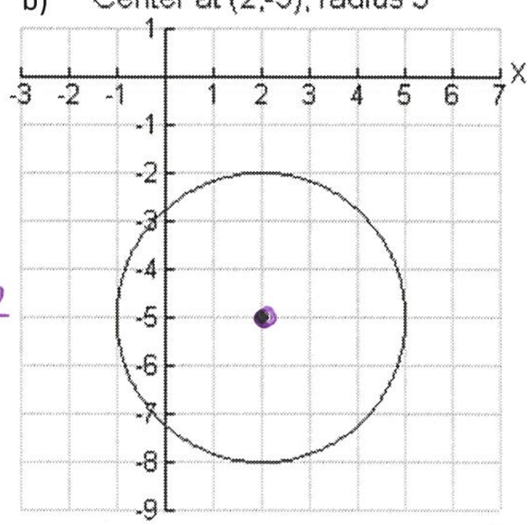
center = (0,0)

radius = 4

$(x-0)^2+(y-0)^2=4^2$

$\boxed{x^2+y^2=16}$

b) Center at (2,-5), radius 3



center = (2,-5)

radius = 3

$(x-2)^2+(y--5)^2=3^2$

$\boxed{(x-2)^2+(y+5)^2=9}$

Now, we are going to "multiply out" the equation from part b.

$$(x - h)^2 + (y - k)^2 = r^2$$

SKIP

$(x-h)(x-h) + (y-k)(y-k) = r^2$

$x^2 - xh - xh + h^2 + y^2 - yk - yk + k^2 = r^2$

$x^2 - 2xh + h^2 + y^2 - 2yk + k^2 = r^2$

When multiplied out, we obtain the "general form" of the equation of a circle. Notice that in this form we can clearly see that the equation of a circle has both x^2 and y^2 terms and these terms have the same coefficient (usually 1).

STANDARD (GENERAL) FORM OF AN EQUATION OF A CIRCLE
$x^2 + y^2 + Dx + Ey + F = 0$

When the equation of a circle appears in "general form", it is often beneficial to convert the equation to "center-radius" form to easily read the center coordinates and the radius for graphing.

1. Convert this equation into center-radius form. State the coordinates of the center of the circle and its radius.

$$x^2 + y^2 - 4x - 6y + 8 = 0$$

This conversion requires use of the technique of completing the square.
We will be creating two perfect square trinomials within the equation.

Steps:

A) Start by grouping the *x* related terms together and the *y* related terms together. Move any numerical constants (plain numbers) to the other side.
B) Get ready to insert the needed values for creating the perfect square trinomials. Remember to balance both sides of the equation.
C) Find each missing value by taking half of the "middle term" and squaring. This value will always be positive as a result of the squaring process.
D) Rewrite in factored form.

$$x^2 + y^2 - 4x - 6y + 8 = 0$$

$-8 \quad -8$

$x^2 - 4x + 4 + y^2 - 6y + 9 = -8 + 4 + 9$

$\left(\frac{-4}{2}\right)^2 \quad \left(\frac{-6}{2}\right)^2$

$\boxed{(x-2)^2 + (y-3)^2 = 5}$

2. Convert this equation into center-radius form. State the coordinates of the center of the circle and its radius.

$$x^2 + y^2 + 2x - 4y - 11 = 0$$

$+11 \quad +11$

$x^2 + 2x + \underline{1} + y^2 - 4y + \underline{4} = 11 + \underline{1} + 4$

$\left(\frac{2}{2}\right)^2 \quad \left(\frac{-4}{2}\right)^2$

$(x+1)^2 + (y-2)^2 = 16$

center = $(-1, 2)$

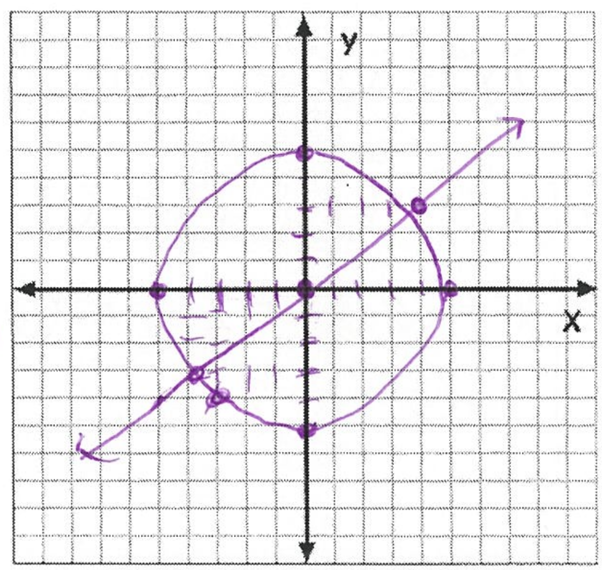
radius = 4

SYSTEMS OF EQUATIONS- CIRCLE AND LINEAR SYSTEMS

Find all the solutions to the following systems of equations algebraically and graphically.

1) $x^2 + y^2 = 25$ center = $(0,0)$ radius = 5

$y = \frac{3}{4}x$ slope = $\frac{3}{4}$ yint = 0



$x^2 + \left(\frac{3}{4}x\right)^2 = 25$

$1x^2 + \frac{9}{16}x^2 = 25$

$\frac{25}{16}x^2 = 25$

$\sqrt{x^2} = \sqrt{16}$

$x = \pm 4$

$x = 4$

$y = \frac{3}{4}(4)$

$y = 3$

$\boxed{(4,3)}$

$x = -4$

$y = \frac{3}{4}(-4)$

$y = -3$

$\boxed{(-4,-3)}$

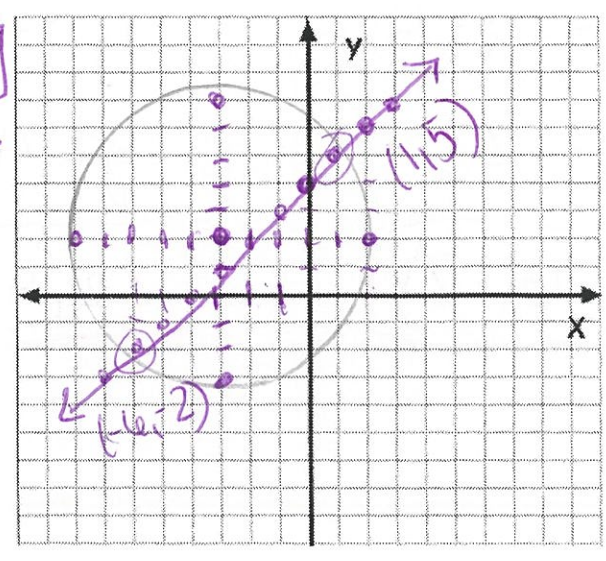
2) $(x + 3)^2 + (y - 2)^2 = 25$ center = $(-3, 2)$ radius = 5

$-2x + 2y = 8$

$+2x \quad +2x$

$2y = 8 + 2x$

$y = 4 + x$ slope = $\frac{1}{1}$ yint = 4



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$(x+3)^2 + (y-2)^2 = 25 \qquad y = x+4$

2

$(x+3)(x+3) + (x+4+2)^2 = 25$

$x^2+3x+3x+9 \qquad (x+2)(x+2)$

$x^2+6x+9+x^2+2x+2x+4 = 25$

$2x^2+10x+13 = 25$
$-25 \quad -25$

$2x^2+10x-12 = 0$

$a \qquad m$

$2(x^2+5x-6) = 0$

$2(x+6)(x-1) = 0$

	-6	1

$x = -6$
$y = x+4$
$y = -6+4$
$y = -2$
$\boxed{(-6,-2)}$

$x = 1$
$y = x+4$
$y = 1+4$
$y = 3$
$\boxed{(1,5)}$