

LESSON #6: EVEN/ODD FUNCTIONS

Do Now:

1. For the following functions, label it odd or even degree and describe the end behavior.

a) $f(x) = x^8 - x$

Even, up + up

b) $f(x) = x^5 + x^4 - 2x^7 + 7$

Odd, up + down

2. What happens to the coordinates (x,y) when its reflected over the y-axis? $\rightarrow (-x, y)$

3. What happens to the coordinates (x,y) when its reflected over the origin? $\rightarrow (-x, -y)$

4. Given: $f(x) = x^6$, evaluate $f(-1) = (-1)^6 = 1$

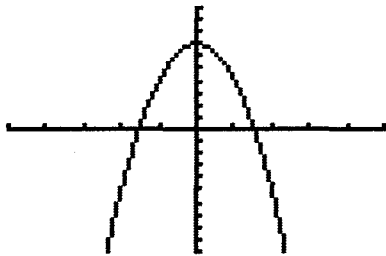
5. Given: $f(x) = x^7$, evaluate $f(-1) = (-1)^7 = -1$

even exponents = positive, odd exponents = negative

We use the term **even function** when a function f satisfies the equation $f(-x) = f(x)$ for every number x in its domain. When we plug in " $-x$ ", the equation does not change.

when you plug in -1 , $f(x)$ does not change

Consider the graph of the function $f(x) = -3x^2 + 7$.



Evaluate $f(-x)$.

$f(-x) = -3(-x)^2 + 7$

$f(-x) = -3x^2 + 7 = f(x)$

$f(x) = f(-x)$ **EVEN**

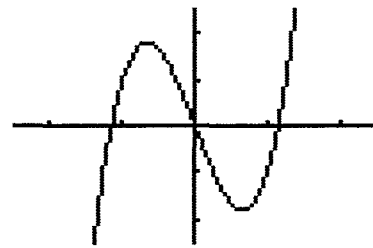
In general, an **EVEN** function has the following properties:

- A) Its graph is symmetric about the y axis.
- B) The exponents of all terms in its equation are even and there can be _____.
- C) $f(-x) = f(x)$

We use the term **odd function** when a function f satisfies the equation $f(-x) = -f(x)$ for every number x in its domain. When we plug in " $-x$ ", all the signs of the equation changes.

when you plug in -1 , $f(x)$ becomes negative

Consider the graph of the function $f(x) = 3x^3 - 4x$.



Evaluate $f(-x)$.

$f(-x) = 3(-x)^3 + 4(-x)$

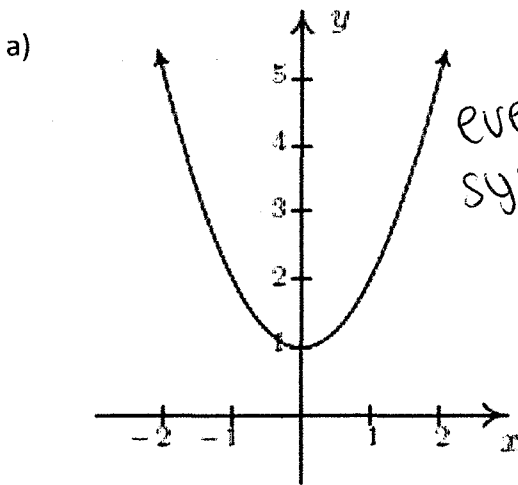
$3 \cdot -x^3 + 4x$

$f(-x) = -3x^3 + 4x = -f(x)$ **ODD**

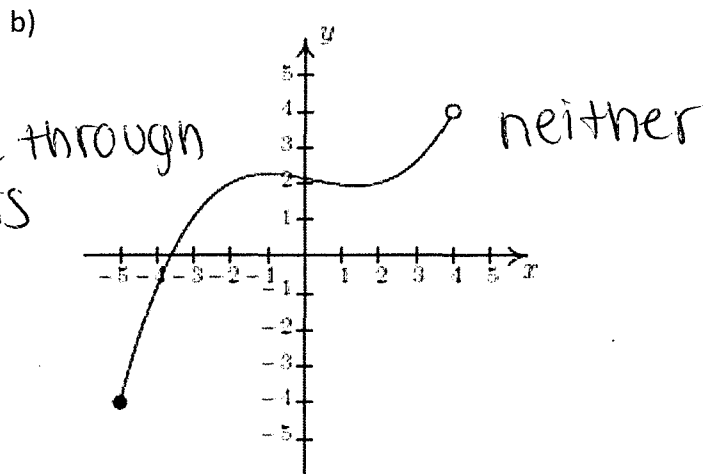
In general, an **ODD** function has the following properties:

- A) Its graph is symmetric about the origin.
- B) The y-intercept is 0
- C) The exponents of all terms in its equation are odd and there can be _____.
- D) $f(-x) = -f(x)$

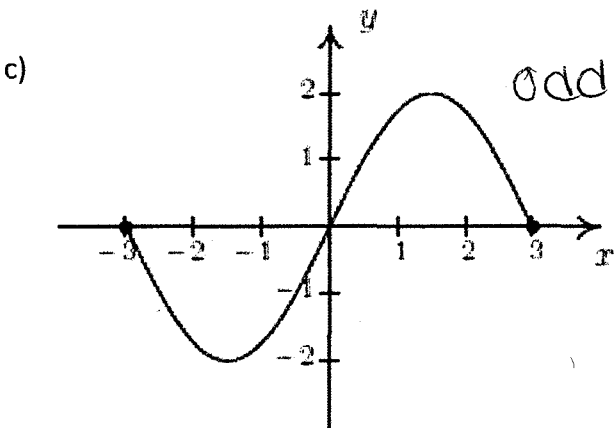
1) Determine if the graphs represent an odd function, an even function, or neither.



even,
symmetric through
y-axis



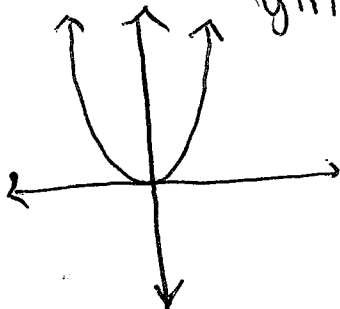
neither



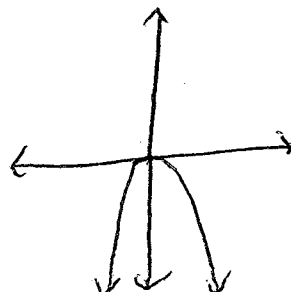
odd, symmetric through origin

2) What happens when you evaluate $f(-x)$ for each of the functions? Sketch a graph for each of the functions.

a) $f(x) = x^4$
 $f(-x) = (-x)^4$
 $f(-x) = x^4$
 EVEN, EB \rightarrow UP, UP
 $y_{int} = 0$



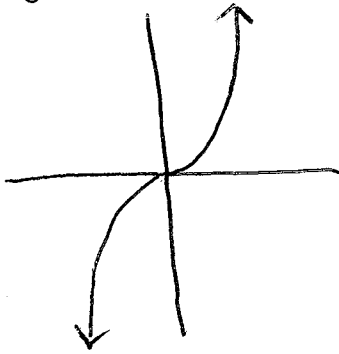
b) $f(x) = -3x^6 - 2x^4$
 $f(-x) = -3(-x)^6 - 2(-x)^4$
 $f(-x) = -3x^6 - 2x^4$
 EVEN, EB \rightarrow DOWN, DOWN
 $y_{intercept} = 0$



3) What happens when you evaluate $f(-x)$ for each of the functions? Sketch a graph for each of the functions.

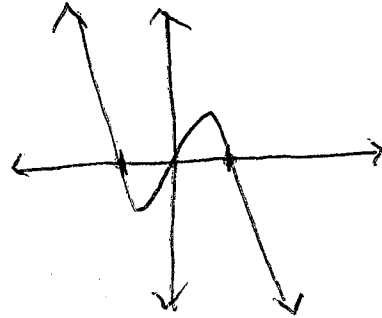
a) $f(x) = x^3$ OPPOSITE!
 $f(-x) = (-x)^3$
 $f(-x) = -x^3$

ODD, EB down, up
 yintercept = 0



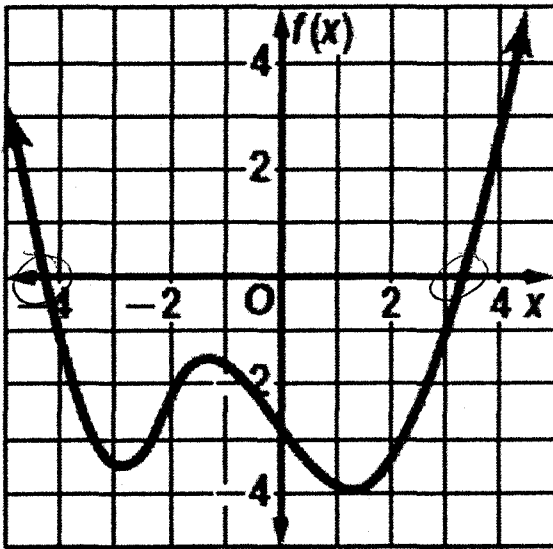
b) $f(x) = -x^5 - 3x^3 + 4x$ OPPOSITE!
 $f(-x) = -(-x)^5 - 3(-x)^3 + 4(-x)$
 $-(-x^5) - 3(-x^3) - 4x$
 $x^5 + 3x^3 - 4x$

Odd, EB UP, down
 yintercept = 0



4) For each graph shown, determine:

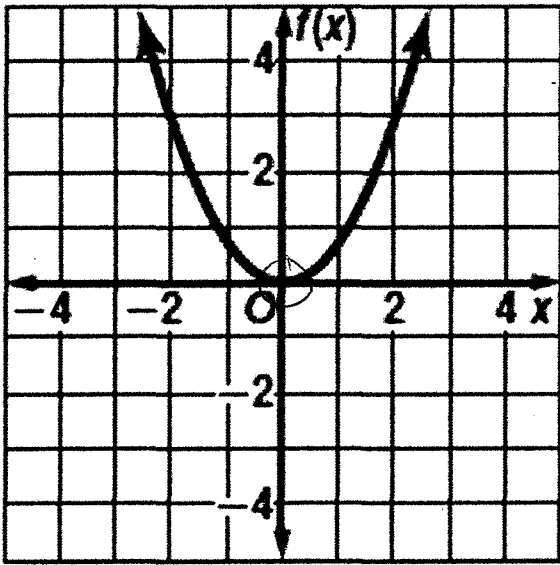
- a) If it represents an odd or even **degree** function
- b) If it represents an odd or even **function** or neither



EB = UP, UP (POSITIVE L.C)

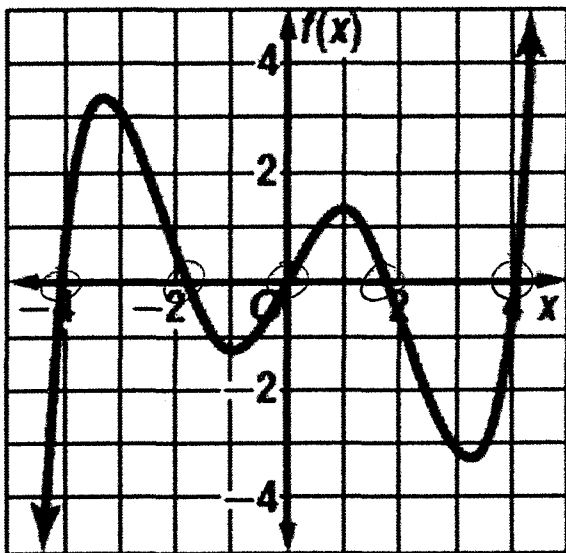
- a) 2 roots, even degree
- b) Neither, no symmetry

5)



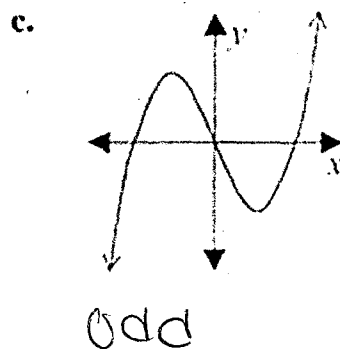
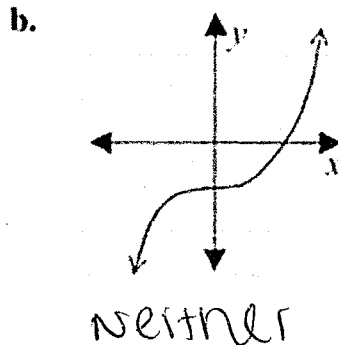
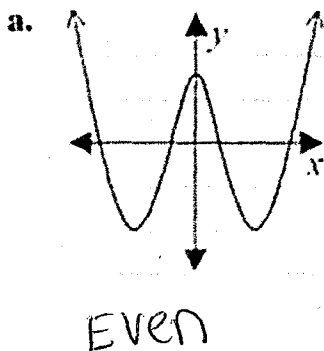
- EB = UP, UP (POSITIVE LC)
- a) 1 root, mult. of 2 → EVEN degree
- b) EVEN, symmetry about y-axis

6)



- EB = down, UP (negative LC)
- a) 5 roots, ODD degree
- b) ODD, symmetry about origin

7. Determine what kind of symmetry, if any, each of the following functions has.



8. Determine what kind of symmetry, if any, each of the following functions has.

a. $f(x) = x^2 - 3$ even, y-axis

b. $f(x) = 2x^3 + x$ odd, origin

c. $f(x) = x^2 - 2x$ neither, none

d. $f(x) = \frac{x}{x^2 - 1}$

9. For each of the following functions, determine if it is even, odd, or neither.

a. $f(x) = x^4 - bx^2 + c$ even

b. $f(x) = x^3 + bx$ odd

For #10-11 WITHOUT THE USE OF YOUR CALCULATOR:

- Determine the degree
- Determine the roots
- Describe the end behavior

10) $y = (x+2)(x-3)(x+1)$

a) 3

b) $\{-2, 3, -1\}$

c) down, up

11) $y = (x-6)^4$

a) 4

b) 6, mult. of 4

c) up; up

12. Solve for x in simplest radical form using any method.

$$5x^2 - 4x - 2 = 0$$

$$a = 5$$

$$b = -4$$

$$c = -2$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(-2)}}{2(5)}$$

$$x = \frac{4 \pm \sqrt{16 + 40}}{10}$$

$$x = \frac{4 \pm \sqrt{56}}{10} \sim \frac{\sqrt{4}}{\sqrt{14}}$$

$$x = \frac{\cancel{4} \pm \cancel{2}\sqrt{14}}{5\cancel{10}}$$

$$x = \frac{2 \pm \sqrt{14}}{5}$$

13. The function $j(x) = x^3 + 9x^2 + 24x + 16$ has a factor of $(x + 1)$. Determine the remaining factors of this function.

$$\begin{array}{r}
 \boxed{x^2 + 8x + 16} \\
 x+1 \overline{) x^3 + 9x^2 + 24x + 16} \\
 \underline{-x^3 x^2} \\
 8x^2 + 24x \\
 \underline{-8x^2 8x} \\
 16x + 16 \\
 \underline{-16x 16} \\
 0
 \end{array}$$

$$\begin{array}{c}
 x^2 + 8x + 16 \\
 (x+4)(x+4)
 \end{array}$$

$$\boxed{\text{FACTORS: } (x+1)(x+4)(x+4)}$$