

**LESSON #5: END BEHAVIOR**

**Do Now:**

a) On the same set of axes, sketch AND label the graphs of  $f(x) = x^2$  and  $f(x) = x^4$ . } one color

b) (DON'T SKETCH) Predict what the graph of  $f(x) = -x^2$  and  $f(x) = -x^4$  look like?

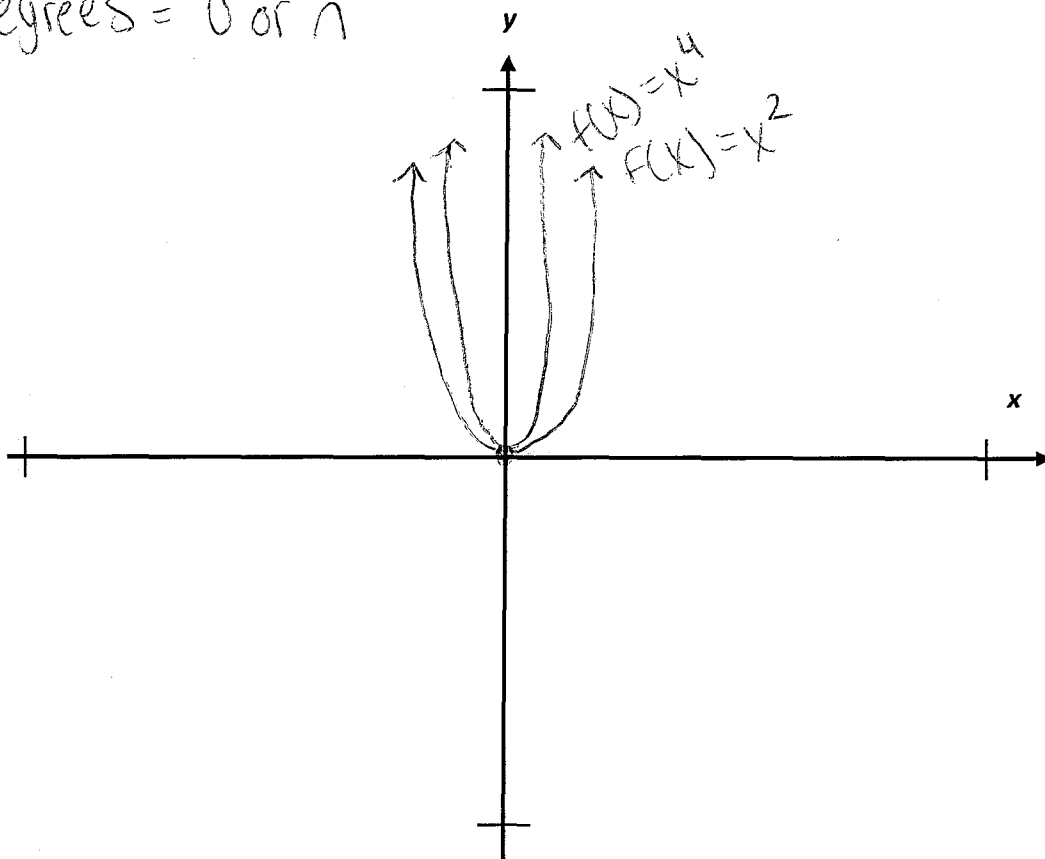
Flipped upside down ↕

c) On the axes shown, sketch AND label the graphs of  $f(x) = x^3$ .

d) On the same axes, sketch AND label  $f(x) = -x^5$  and  $f(x) = x^7$

e) Describe some similarities and differences between functions that have even and odd degrees.

even degrees = U or ∩



Use your calculator to graph each polynomial function below. Fill in the table paying attention to how the function enters and exits the view screen of the calculator.

	Function	Even or Odd Degree	Positive or Negative Leading Coefficient	Rise or Fall to the left	Rise or Fall to the right
1)	$f(x) = 2x^2 - 3x + 3$	Even	+	Rise	Rise
2)	$f(x) = -x^4 + x^3 - x^2 + 3$	Even	-	Fall	Fall
3)	$f(x) = x^5 - x^3 + 2x + 6$	Odd	+	Fall	Rise
4)	$f(x) = -x^3 + 2x^2 - x - 4$	Odd	-	Rise	Fall

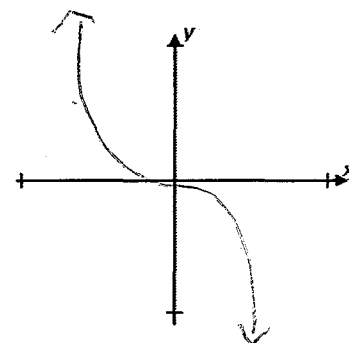
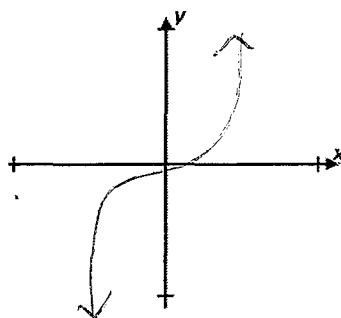
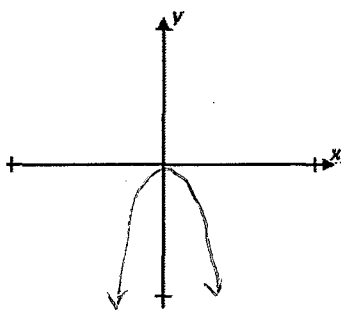
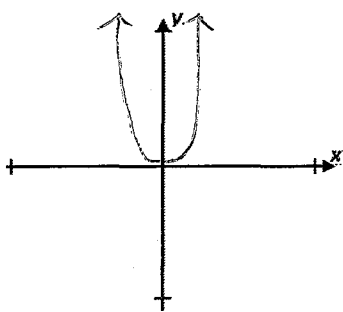
Sketch each of the 4 functions:

1) Even with + coefficient

2) Even with - coefficient

3) Odd with + coefficient

4) Odd with - coefficient



**End Behavior (description):** Let  $f$  be a function whose domain and range are subsets of the real numbers. The *end behavior* of a function  $f$  is a description of what happens to the values of the function:

- as  $x$  approaches infinity ( $\infty$ ),  $f(x) \rightarrow +\infty$  and
- as  $x$  approaches infinity ( $\infty$ ),  $f(x) \rightarrow -\infty$

**End behavior of Polynomial Functions:**

**EVEN DEGREE** → Left and Right End Behavior is the same

**ODD DEGREE** → Left and Right End Behavior is opposite

Find the end behavior of the function:

1)  $f(x) = x^4 - 4x^3 + 3x + 25$

Degree: 4

Leading Coefficient: 1

End Behavior: as  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$   
as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow \infty$

2)  $f(x) = -x^3 + 5x^2 - 1$

Degree: 3

Leading Coefficient: -1

End Behavior: as  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$   
as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow \infty$

3)  $f(x) = -2x^6 + 7x^3 + 3x^2$

Degree: 6

Leading Coefficient: -2

End Behavior: as  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$   
as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow -\infty$

\* 4)  $f(x) = -4x^4 + 5x^5 + x^2$

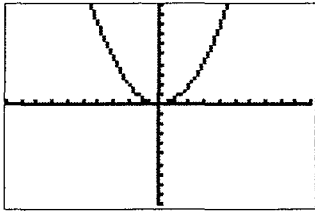
Degree: 5

Leading Coefficient: +5

End Behavior: as  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$   
as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow -\infty$

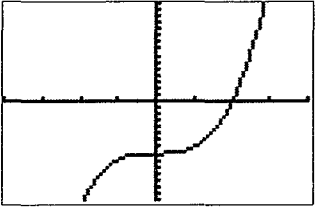
5) Without using a calculator, match each graph below in column 1 with the function in column 2 that it represents.

a.



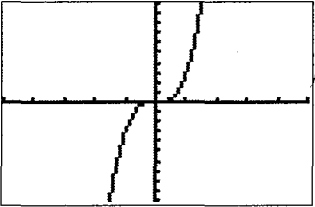
1.  $y = 3x^3$

b.



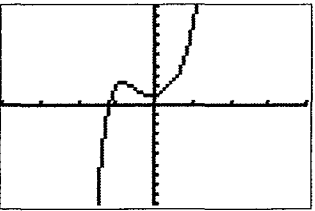
2.  $y = \frac{1}{2}x^2$

c.



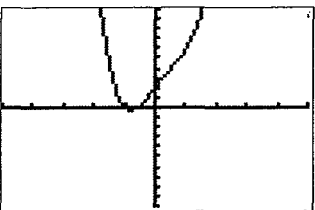
3.  $y = x^3 - 8$

d.



4.  $y = x^4 - x^3 + 4x + 2$

e.



5.  $y = 3x^5 - x^3 + 4x + 2$

LAB #6: POLYNOMIAL DEGREE AND END BEHAVIOR

For #'s 1-5, write the degree and determine the end behavior (UP/DOWN) of each polynomial without a calculator.

<p>1) <math>f(x) = -2x^3 + x</math></p> <p>Degree: <u>3</u></p> <p>As <math>x \rightarrow -\infty</math>, graph goes <u>UP</u></p> <p>As <math>x \rightarrow +\infty</math>, graph goes <u>DOWN</u></p>	<p>2) <math>f(x) = 3x^2 + 5x - 4</math></p> <p>Degree: <u>2</u></p> <p>As <math>x \rightarrow -\infty</math>, graph goes <u>UP</u></p> <p>As <math>x \rightarrow +\infty</math>, graph goes <u>UP</u></p>
<p>3) <math>f(x) = (x-1)(x^4 + 2x^3 - 3x^2 + 10)</math></p> <p>Degree: <u>5</u></p> <p>As <math>x \rightarrow -\infty</math>, graph goes <u>DOWN</u></p> <p>As <math>x \rightarrow +\infty</math>, graph goes <u>UP</u></p>	<p>4) <math>f(x) = -x^4 - 4x^2 + 1</math></p> <p>Degree: <u>4</u></p> <p>As <math>x \rightarrow -\infty</math>, graph goes <u>DOWN</u></p> <p>As <math>x \rightarrow +\infty</math>, graph goes <u>DOWN</u></p>

5. Consider the function  $f(x) = x^3 - 13x^2 + 44x - 32$ .

a. Use the fact that  $x - 4$  is a factor of  $f$  to factor this polynomial.

①

$$\begin{array}{r}
 \overline{x^2 + 9x + 8} \\
 x - 4 \overline{) x^3 - 13x^2 + 44x - 32} \\
 \underline{-x^3 + 4x^2} \phantom{+ 44x - 32} \\
 -9x^2 + 44x \phantom{- 32} \\
 \underline{+9x^2 - 36x} \phantom{- 32} \\
 8x - 32 \\
 \underline{-8x + 32} \\
 0
 \end{array}$$

②

$$\begin{array}{l}
 x^2 + 9x + 8 \\
 (x + 8)(x + 1)
 \end{array}$$

$$\boxed{f(x) = (x + 1)(x - 4)(x + 8)}$$

b. Find the  $x$ -intercepts for the graph of  $f$ .

$$\begin{array}{c}
 f(x) = (x + 1)(x - 4)(x + 8) \\
 \hline
 -1 \quad | \quad 4 \quad | \quad -8
 \end{array}$$

$$\{-8, -1, 4\}$$

6. The Center for Transportation Analysis (CTA) studies all aspects of transportation in the United States, from energy and environmental concerns to safety and security challenges. A 1997 study compiled the following data of the fuel economy in miles per gallon (mpg) of a car or light truck at various speeds measured in miles per hour (mph). The data is compiled in the table below.

- A) This data can be modeled by a polynomial function. Determine if the function that models the data would have an even or odd degree.

*even EB low, low*

Speed (mph)	Fuel Economy (mpg)
15	24.4
20	27.9
25	30.5
30	31.7
35	31.2
40	31.0
45	31.6
50	32.4
55	32.4
60	31.4
65	29.2
70	26.8
75	24.8

- B) Is the leading coefficient of the polynomial that can be used to model this data positive or negative?

*negative*