Key Date 5/7/18

REVEIEW: SEQUENCE AND SERIES

*****TEST- WEDNESDAY 5/9/18*****

1) Determine whether each sequence is an arithmetic or geometric sequence. State the common difference or common ratio. State if the sequence would be linear or exponential.

Sequence	Identify if the sequence is Arithmetic or Geometric (A or G)	Identify the Common difference or the Common ratio (D or R)	Identify if the sequence will model a Linear or Exponential function (L or E)
a) 4, 7, 10, 13, ...	A	$d = 3$	L
b) 15, 13, 11, 9, ...	A	$d = -2$	L
c) 1, 4, 16, 64, ...	G	$r = 4$	E
d) 2, -4, 8, -16, ...	G	$r = -2$	E

2) Determine the 10th term of the arithmetic sequence in which $a_1 = 18$ and $a_7 = -6$.

↳ slope! $\frac{-6-18}{7-1} = \frac{-24}{6} = -4 = d$

$a_1 = 18$

$a_n = a_1 + d(n-1)$

$a_n = 18 - 4(n-1)$

$a_n = 18 - 4n + 4$

$\boxed{a_n = 22 - 4n}$

$a_{10} = 22 - 4(10)$

$\boxed{a_{10} = -18}$

3) Determine the common ratio of a geometric sequence for which $a_1 = 2$ and $a_5 = 2592$.

$a_n = a_1 r^{n-1}$

$a_5 = 2r^{5-1}$

$\frac{2592}{2} = \frac{2r^4}{2}$

$\sqrt[4]{1296} = \sqrt[4]{r^4}$

$\boxed{r = \pm 6}$

↳ use calc! ExpReg

$y = .33(6)^x$

$\boxed{r = 6}$

4) A recursive formula for the sequence 18, 9, 4.5, ... is

GEOMETRIC! $r = \frac{1}{2}$

(1) $g_1 = 18$

$g_n = \frac{1}{2} g_{n-1}$

(3) $g_1 = 18$

$g_n = 2g_{n-1}$

~~(2)~~ $g_n = 18\left(\frac{1}{2}\right)^{n-1}$

~~(4)~~ $g_n = 18(2)^{n-1}$

Not recursive! needs to start w/ first term

5) Determine the first four terms of the recursive sequence defined below.

$a_1 = 4$

$a_n = 3 \cdot a_{n-1}$

$a_2 = 3a_{2-1}$ $\quad$ $a_3 = 3a_{3-1}$ $\quad$ $a_4 = 3a_{4-1}$

$a_2 = 3a_1$ $\quad$ $a_3 = 3a_2$ $\quad$ $a_4 = 3a_3$

$a_2 = 3(4)$ $\quad$ $a_3 = 3(12)$ $\quad$ $a_4 = 3(36)$

$a_2 = 12$ $\quad$ $\boxed{a_3 = 36}$ $\quad$ $a_4 = 108$

$\{4, 12, 36, 108\}$

6) Given the following sequence following: -12, -7, -2, 3.....

a. Write a *recursive formula* for the sequence.

arithmetic! $\boxed{a_1 = -12 \quad a_n = 5 + a_{n-1}}$

b. Write an *explicit formula* for the sequence.

$a_n = a_1 + d(n-1)$

$\boxed{a_n = -12 + 5(n-1)}$

c. Use the explicit formula to find the 17th term.

$a_{17} = -12 + 5(17-1)$

$a_{17} = -12 + 5(16)$

$a_{17} = -12 + 80$

$\boxed{a_{17} = 68}$

7) Given the following sequence 16, 32, 64, 128......

a. Write a *recursive formula* for the sequence.

Geometric! $\boxed{a_1 = 16 \quad a_n = 2a_{n-1}}$

b. Write the *explicit formula for the sequence.*

$a_n = a_1 r^{n-1}$

$\boxed{a_n = 16(2)^{n-1}}$

c. Use the explicit formula to find the 17th term.

$a_{17} = 16(2)^{17-1}$

$a_{17} = 16(2)^{16}$

$\boxed{a_{17} = 1,048,576}$

For #8 & 9, evaluate the expression.

8) $\sum_{n=1}^{5}(4n+1)$

$4(1)+1 = 5$

$4(2)+1 = 9$

$4(3)+1 = 13$

$4(4)+1 = 17$

$4(5)+1 = +21$

$\boxed{65}$

9) $3\sum_{n=1}^{3} 2^{n-1}$

$2^{1-1} = 2^0 = 1$

$2^{2-1} = 2^1 = 2$

$2^{3-1} = 2^2 = +4$

$3 \times 7 =$

$\boxed{21}$

10) Which summation represents $\underset{a_1}{5} + 7 + 9 + 11 + \ldots + 43$? arithmetic, $d=2$

~~A.~~ $\sum_{n=5}^{43} n$ B. $\sum_{n=1}^{20} (2n+3)$ C. $\sum_{n=4}^{24} (2n-3)$ D. $\sum_{n=3}^{23} (3n-4)$

$a_n = 5 + 2(n-1)$
$a_n = 5 + 2n - 2$
$a_n = 2n + 3$

11) Write the following using sigma notation: $\underbrace{2 + 5}_{+3} + \underbrace{8}_{+3} + \underbrace{11}_{+3} + 14 + \ldots + 59$

$\boxed{\sum_{n=1}^{20} 3n-1}$

arithmetic, $d=3$

$a_n = 2 + 3(n-1)$
$a_n = 2 + 3n - 3$
$a_n = 3n - 1$

what term #?

$59 = 3n - 1$
$+1 \quad +1$
$\frac{60}{3} = \frac{3n}{3}$
$n = 20$

12) The formula below can be used to model which scenario?

$a_1 = 3000$
$a_n = 0.80a_{n-1}$ → exp. decay

~~(1)~~ The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it. linear

~~(2)~~ The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it. linear

~~(3)~~ A bank account starts with a deposit of \$3000, and each year it grows by 80%.

(4) The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

13) Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

(1) \$11,622,614.67
(2) \$17,433,922.00
~~(3)~~ \$116,226,146.80
(4) \$1,743,392,200.00

a_1 ↓ 1, 3, 9, 27

Geometric
$r = 3$

~~$a_{20} = 1(3)^{20-1}$~~
~~calc~~

series!

$\sum_{1}^{20} 1(3)^{n-1}$ OR $S_{20} = \frac{1 - 1(3)^{20}}{1-3}$

use calculator!

14) Katie invests $5,000 in an account that pays 1.65% annual interest compounded *continuously.* How many years, to the nearest tenth, will it take for Katie's investment to double?

PERT!

$\frac{10{,}000}{5000} = \frac{5000e^{.0165t}}{5000}$

$2 = e^{.0165t}$

$\ln 2 = .0165t$

$t = 42.0082$

$\boxed{t = 42 \text{ years}}$

15) Given $f(9) = -2$, which function can be used to generate the sequence $-8, -7.25, -6.5, -5.75, \ldots$?

(1) $f(n) = -8 + 0.75n$

~~(2)~~ $f(n) = -8 - 0.75(n - 1)$

(3) $f(n) = -8.75 + 0.75n$

(4) $f(n) = -0.75 + 8(n - 1)$

substitute 9 for n

$= -2$ ✓

$d = .75$

$a_1 = -8$

$a_n = -8 + .75(n-1)$

$-8 + .75n - .75$

$\boxed{-8.75 + .75n}$

16) The sequence $a_1 = 6$, $a_n = 3a_{n-1}$ can also be written as

~~(1)~~ $a_n = 6 \cdot 3^n$

~~(2)~~ $a_n = 6 \cdot 3^{n+1}$

(3) $a_n = 2 \cdot 3^n$

~~(4)~~ $a_n = 2 \cdot 3^{n+1}$

Guess & check!

geometric

$r = 3$

$a_1 = 6$

$a_n = 6(3)^{n-1}$

17) The population of Jamesburg for the years 2010 – 2013, respectively, was reported as follows:

250,000 250,937 251,878 252,822

exponential- Geometric

$r = 1.003748$ $\frac{a_n}{a_{n-1}}$

How can this sequence be recursively modeled?

~~(1)~~ $j_n = 250{,}000(1.00375)^{n-1}$

~~(2)~~ $j_n = 250{,}000 + 937^{(n-1)}$

} Not recursive

(3) $j_1 = 250{,}000$
$j_n = 1.00375 j_{n-1}$

(4) $j_1 = 250{,}000$
$j_n = j_{n-1} + 937$ → linear!

18) Write the following using sigma notation: $1 + 3 + 9 + 27 + \ldots\ldots 729$

$\underbrace{1+3}_{x3}\underbrace{+9}_{x3}\underbrace{+27}_{x3}$

→ what term #?

$729 = 3^{n-1}$

$\log_3 729 = n-1$

$+1 \quad +1$

$n = 7$

$$\sum_{n=1}^{7} 1(3)^{n-1}$$

19) Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed \$2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed \$6.25 to replace his library card and pay the fine for the overdue book.

Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for a_n, the nth term of this sequence.

$a_5 = 2.25$

$a_{21} = 6.25$

$\frac{6.25-2.25}{21-5} = \frac{4}{16} = \frac{1}{4} = d$

$\underset{a_1}{1.25} \quad \underset{a_2}{1.5} \quad \underset{a_3}{1.75} \quad \underset{a_4}{2} \quad \underset{a_5}{2.25}$

OR use STAT CALC Lin Reg!

$a_n = 1.25 + .25(n-1)$

$1.25 + .25n - .25$

$\boxed{a_n = 1 + .25n}$

Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

$a_{60} = 1 + .25(60)$

$\boxed{a_{60} = 16}$

$1 - .04 = .96$

20) There are 50,000 deer in an area and the population is decreasing at a rate 4% in each successive year.

a) Write a geometric series formula, S_n, for the total number of deer over n years.

$$S_n = \frac{a_1 - a_1 r^{n-1}}{1-r}$$

$$S_n = \frac{50000 - 50000(.96)^n}{1-.96}$$

b) Use this formula to find the total number of deer in 13 years, rounded to the *nearest integer*.

$$S_{13} = \frac{50000 - 50000(.96)^{13}}{1-.96}$$

$S_{13} = 514748.2912 \approx \boxed{514{,}748 \text{ deer}}$

21) Rachel and Marc were given the information shown below about the bacteria growing in a Petri dish in their biology class.

Time, hour, (x)	0	1	2	3	4	5
Population (y)	250	330	580	800	1650	3000

+80
x 1.32

Rachel wants to model this information with a linear function. Marc wants to use an exponential function.

a) Which model is the better choice? Explain why you chose this model.

Exponential b/c it does not increase by a constant rate.

b) Based on the model you chose in part (a), write the equation that represents this data, rounding all values to the *nearest hundredth.*

$y = 215.98(1.65)^x$

c) Using your equation in part (b), predict how many bacteria will be present after 10 hours.

$x=10$

$y = 215.98(1.65)^{10}$

$y = 32303.75 \approx \boxed{32303 \text{ bacteria}}$

d) Using your equation in part (b), predict the number of hours, to the nearest tenth, it will take for the number of bacteria to reach 500,000.

$y=500{,}000$

$\frac{500{,}000}{215.98} = \frac{\cancel{215.98}(1.65)^x}{\cancel{215.98}}$

$2315.0291 = 1.65^x$

$\log_{1.65} 2315.0291 = x$

$\boxed{x = 15.5 \text{ hrs}}$

22) A gymnast performs a series of full circle swings during a high bar routine. The distance that the gymnast's feet are above the ground at a given time, t in seconds, can be modeled by the equation: $f(t) = -85\cos\left(\frac{8\pi}{5}t\right) + 109$.

a) What is the midline of this function?

$\boxed{y = 109}$

b) What is the amplitude of this function?

$\boxed{85}$

c) State the range of the distance of the gymnast's feet from the ground.

$[109-85, 109+85] = \boxed{[24, 194]}$

d) What is the period of this function? Explain what this value means in regards to the gymnast's swing.

$\frac{2\pi}{\frac{8\pi}{5}} = \frac{5}{4} = \boxed{1.25}$ it takes 1.25 seconds to complete 1 full circle