Name: Key

Date: 5/1/18

CC ALGEBRA 2

TROICI

LESSON#2: EXPLICIT VS RECURSIVE FORMULAS

Do Now:

1) Given the explicit formula: $a_n = 2(3)^{n-1}$

   a) Identify the common ratio. 3

   b) Using the explicit formula, find the first three terms.

$\boxed{\{2, 6, 18\}}$

$a_1 = 2(3)^{1-1}$

$a_1 = 2(3)^0$

$a_1 = \textcircled{2}$

$a_2 = 2(3)^{2-1}$

$a_2 = 2(3)^1$

$a_2 = \textcircled{6}$

$a_3 = 2(3)^{3-1}$

$a_3 = 2(3)^2$

$a_3 = \textcircled{18}$

2) A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern.

   a) Identify a_1 60

   b) Find the common difference. 8 = d

   c) If the theater has 10 rows, how many seats are in the 10th row? Show all work.

$a_n = 60 + 8(n-1)$

$a_{10} = 60 + 8(10-1)$

$a_{10} = 60 + 8(9)$

$a_{10} = 60 + 72 = \boxed{132}$

Arithmetic Formula to find the n^{th} term

$$a_n = a_1 + (n-1)d$$

a_1 → First term

a_n → nth term

n → number of terms

d → common difference

Geometric Formula to find the n^{th} term

$$a_n = a_1 r^{n-1}$$

r = common ration

$-6 \quad -6$

1) Given the arithmetic sequence: 32, 26, 20, 14, 8, ...

$-6 \quad -6$

a) Write the explicit formula.

$a_n = a_1 + d(n-1)$

$a_n = 32 + -6(n-1)$

$a_n = 32 - 6n + 6 = \boxed{38-6n}$

b) Use the explicit formula to find the 20th term.

$a_{20} = 38 - 6(20)$

$\boxed{a_{20} = -82}$

2) Given the geometric sequence 160, 80, 40 .....

$\div 2 \div 2$

a) Write the explicit formula

$a_n = a_1 r^{n-1}$

$a_n = 160\left(\frac{1}{2}\right)^{n-1}$

b) Use the explicit formula to find the 8th term.

$a_8 = 160\left(\frac{1}{2}\right)^{8-1}$

$\boxed{a_8 = 1.25}$

3) In an arithmetic sequence, $a_4 = 19$ and $a_7 = 31$. Determine a formula for a_n, the n^{th} term of this sequence.

$\frac{31-19}{7-4} = \frac{12}{3} = \boxed{4 = d}$

$\boxed{\frac{7}{a_1}}, \frac{11}{a_2}, \frac{15}{a_3}, \frac{19}{a_4}$

Need a_1 and r!

$a_n = 7 + 4(n-1)$

$a_n = 7 + 4n - 4$

$\boxed{a_n = 3 + 4n}$

4) The third term of a geometric sequence is 3 and the sixth term is $\frac{1}{9}$. Find the first term.

$a_3 = 3 \rightarrow 3 = a_1 r^{3-1} \Rightarrow 3 = a_1 r^2$

$a_6 = \frac{1}{9} \rightarrow \frac{1}{9} = a_1 r^{6-1} \Rightarrow \frac{1}{9} = a_1 r^5$

solve for a_1: $\frac{3}{r^2} = \frac{a_1 \cancel{r^2}}{\cancel{r^2}} \Rightarrow a_1 = \frac{3}{r^2}$

$\frac{\frac{1}{9}}{r^5} = \frac{a_1 \cancel{r^5}}{\cancel{r^5}} \qquad a_1 = \frac{\frac{1}{9}}{r^5}$

set = to each other

$\frac{3}{r^2} = \frac{\frac{1}{9}}{r^5}$

$\frac{3r^5}{r^2} = \frac{\frac{1}{9}\cancel{r^2}}{\cancel{r^2}}$

$\frac{\cancel{3}r^3}{\cancel{3}} = \frac{\frac{1}{9}}{3}$

$\sqrt[3]{r^3} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}}$

$\boxed{r = \frac{1}{3}}$

$\boxed{\frac{27}{a_1}}, \frac{9}{a_2}, \frac{3}{a_3}$

5) Find the number of terms in the following sequence: 7,10,13,.....,55

$+3 \quad +3$

$a_n = a_1 + d(n-1)$

$a_n = 7 + 3(n-1)$

$55 = 7 + 3(n-1)$

$48 = 3n - \cancel{3}$

$+3 \qquad +\cancel{3}$

$51 = 3n$

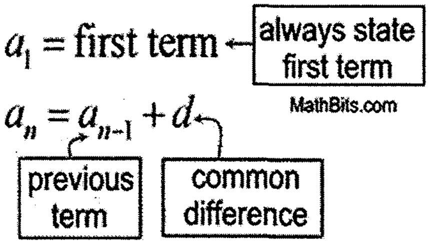
$\boxed{n = 17}$

RECURSIVE FORMULAS

Recursive Formulas

- Dependent on the previous term to develop a pattern.
- Gives you the n^{th} term of a sequence using the term before, *n-1*.

a_1 = the first term in the sequence
a_n = the n^{th} term in the sequence
a_{n-1} = the term before the n^{th} term
n = the term number
d = the common difference.



1. Find the first 4 terms of the sequence using the recursive definition.

$a_1 = -2$
$a_n = a_{n-1} + 3$

$a_2 = a_{2-1} + 3$
$a_2 = a_1 + 3$
$a_2 = (-2) + 3$
$a_2 = 1$

$a_3 = a_{3-1} + 3$
$a_3 = a_2 + 3$
$a_3 = (1) + 3$
$a_3 = 4$

$a_4 = a_{4-1} + 3$
$a_4 = a_3 + 3$
$a_4 = 4 + 3$
$a_4 = 7$

$\{-2, 1, 4, 7\}$

2. Write the first 5 terms of the recursive sequence:

$a_1 = 4$
$a_n = a_{n-1} - 5$

$a_2 = a_1 - 5$
$a_2 = 4 - 5 = -1$

$a_3 = a_2 - 5$
$a_3 = -1 - 5 = -6$

$a_4 = a_3 - 5$
$a_4 = -6 - 5 = -11$

$a_5 = a_4 - 5$
$a_5 = -11 - 5 = -16$

$\{4, -1, -6, -11, -16\}$

3. Write the first 4 terms of the recursive sequence:

$a_1 = 12$
$a_{n+1} = a_n + 2$

$a_2 = a_1 + 2$
$a_2 = 12 + 2 = 14$

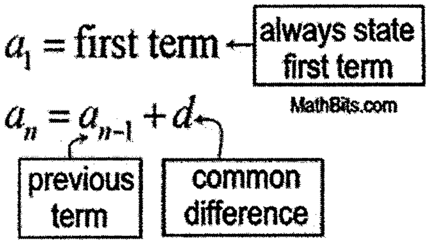
$a_3 = a_2 + 2$
$a_3 = 14 + 2 = 16$

$a_4 = a_3 + 2$
$a_4 = 16 + 2 = 18$

$\{12, 14, 16, 18\}$

To write a recursive formula for an arithmetic sequence:

- Determine if the sequence is arithmetic
- Find the common difference.
- Create a recursive formula by stating the first term, and then stating the formula to be the previous term plus the common difference.



4. State *recursive formula* for this sequence: 32, 38, 44, 50, …

+6 +6 +6

$$\boxed{\begin{aligned} a_1 &= 32 \\ a_n &= a_{n-1} + 6 \end{aligned}}$$

5. Consider the sequence following: 35, 30, 25, 20, 15, 10, …

-5 -5

a) Write a *recursive formula* for the sequence.

$$\boxed{\begin{aligned} a_1 &= 32 \\ a_n &= a_{n-1} - 5 \end{aligned}}$$

b) Write an *explicit formula* for the sequence.

$a_n = a_1 + d(n-1)$

$a_n = 35 - 5(n-1)$

$a_n = 35 - 5n + 5$

$\boxed{a_n = 40 - 5n}$

c) Find the 18th term. Which formula is easier to use? Explain why?

Explicit b/c it is terms of n

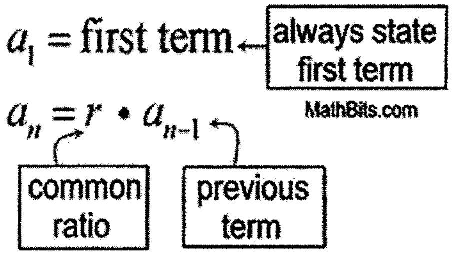
$a_{18} = 40 - 5(18) = \boxed{-50}$

To write a recursive formula for a geometric sequence:

1. Determine if the sequence is geometric .

2. Find the common ratio.

3. Create a recursive formula by stating the first term, and then stating the formula to be the common ratio times the previous term.



x3 x3

7. Consider the sequence following: 3, 9, 27, 81…

a) Write a *recursive formula* for the sequence.

$a_1 = 3$

$a_n = 3 \cdot a_{n-1}$

b) Write an *explicit formula* for the sequence.

$a_n = 3(3)^{n-1}$

c) Use the explicit formula to find the 10th term.

$a_{10} = 3(3)^{10-1}$

$a_{10} = 3(3)^9$

$\boxed{a_{10} = 59049}$

Practice Problems:

8) What is the third term of the recursive sequence below?

$a_1 = -6$

$a_n = \frac{1}{2}a_{n-1} - n$

$a_2 = \frac{1}{2}a_1 - 2$

$a_2 = \frac{1}{2}(-6) - 2$

$a_2 = -3 - 2 = \boxed{-5}$

$a_3 = \frac{1}{2}a_2 - 3$

$a_3 = \frac{1}{2}(-5) - 3$

$a_3 = -2.5 - 3$

$a_3 = \boxed{-5.5}$

9) What is the fourth term of the sequence defined by $a_1 = 3xy^5$

$a_n = \left(\frac{2x}{y}\right)a_{n-1}$?

$a_2 = \left(\frac{2x}{y}\right)a_1$

$a_2 = \left(\frac{2x}{y}\right)\left(\frac{3xy^5}{1}\right)$

$a_2 = \frac{6x^2y^5}{y}$

$\boxed{a_2 = 6x^2y^4}$

$a_3 = \left(\frac{2x}{y}\right)a_2$

$a_3 = \left(\frac{2x}{y}\right)\left(\frac{6x^2y^4}{1}\right)$

$a_3 = \frac{12x^3y^4}{y}$

$a_3 = \boxed{12x^3y^3}$

$a_4 = \left(\frac{2x}{y}\right)a_3$

$a_4 = \left(\frac{2x}{y}\right)\left(\frac{12x^3y^3}{1}\right)$

$a_4 = \frac{24x^4y^3}{y}$

$\boxed{a_4 = 24x^4y^2}$

10) Find the third term in the recursive sequence: $a_{k+1} = 2a_k - 1$, where $a_1 = 3$.

$a_{1+1} = 2a_1 - 1$

$a_2 = 2(3) - 1 = \boxed{5}$

$a_{2+1} = 2a_2 - 1$

$a_3 = 2(5) - 1$

$\boxed{a_3 = 9}$