

LESSON #7: MORTGAGE FORMULAS

1. Monthly mortgage payments can be found using the formula below:

$$M = \frac{P \left(\frac{r}{12} \right) \left(1 + \frac{r}{12} \right)^n}{\left(1 + \frac{r}{12} \right)^n - 1}$$

M = monthly payment

P = amount borrowed

r = annual interest rate

n = number of monthly payments

The Banks family would like to borrow \$120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the fewest number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than \$720.

$$720 \geq \frac{120,000 \left(\frac{.048}{12} \right) \left(1 + \frac{.048}{12} \right)^n}{\left(1 + \frac{.048}{12} \right)^n - 1}$$

$\xrightarrow{\text{calc}}$ $\xrightarrow{\text{calc}}$ want: n

$$\frac{720}{1} \geq \frac{480 (1.004)^n}{(1.004)^n - 1}$$

$$480 (1.004)^n \geq 720 ((1.004)^n - 1)$$

$$480 (1.004)^n \geq 720 (1.004)^n - 720$$

$$-720 (1.004)^n \geq -720 (1.004)^n - 720$$

$$\frac{-240 (1.004)^n}{-240} \geq \frac{-720}{-240}$$

$$(1.004)^n \geq 3$$

$$\log_{1.004} 3 = n \rightarrow n = 275.2020 \text{ months}$$

$$\div 12 = \boxed{23 \text{ years}}$$

Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of \$21,000 and a \$1000 down payment, to the nearest cent.

2a.

$$i = .00625$$

$$P_n = 21000 - 1000$$

$$P_n = 20,000$$

$$P_n = PMT \left(\frac{1 - (1+i)^{-n}}{i} \right)$$

$$P_n = \text{present amount borrowed } 20,000$$

$$n = \text{number of monthly pay periods } 5 \times 12 = 60 \text{ months}$$

$$PMT = \text{monthly payment } ?$$

$$i = \text{interest rate per month } .00625$$

$$20000 = PMT \left(\frac{1 - (1 + .00625)^{-60}}{.00625} \right)$$

calc

$$\frac{20000}{49.9053} = PMT \frac{(49.9053)}{49.9053}$$

$$\boxed{PMT = \$400.76}$$

2b.

The affordable monthly payment is \$300 for the same time period. Determine an appropriate down payment, to the nearest dollar. PMT

$$21000 - x = 300 \left(\frac{1 - (1 + .00625)^{-60}}{.00625} \right)$$

$$21000 - x = 14971.5924$$

$$-21000 \quad -21000$$

$$-x = -6028.4075$$

$$x = 6028.41$$

$$\boxed{x = \$6028 \text{ down payment}}$$

MORE PRACTICE!

Jim is looking to buy a vacation home for $\$172,600$ near his favorite southern beach. The formula to compute a mortgage payment, M , is $M = P \cdot \frac{r(1+r)^N}{(1+r)^N - 1}$ where P is the principal amount of the loan, r is the monthly interest rate, and N is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage.

$r = .00305\%$ $N = 15 \times 12 = 180 \text{ month}$

With no down payment, determine Jim's mortgage payment, rounded to the nearest dollar.

want: m

$$M = 172,600 \cdot \left(\frac{.00305(1 + .00305)^{180}}{(1 + .00305)^{180} - 1} \right)$$

calc

$$m \approx 172,600 \cdot .007227$$

$$m = 1247.49$$

$$m = \$1247$$

Algebraically determine and state the down payment, rounded to the nearest dollar, that Jim needs to make in order for his mortgage payment to be $\$1100$.

m

$$1100 = 172,600 - x \cdot \left(\frac{.00305(1 + .00305)^{180}}{(1 + .00305)^{180} - 1} \right)$$

$$\frac{1100}{.007227} = \frac{(172,600 - x) (.007227)}{.007227}$$

$$152193.191 = 172,600 - x$$

$$\begin{array}{r} -172,600 \quad -172,600 \\ \hline -20406.8089 = -x \end{array}$$

$$x = \$20407$$