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CC ALGEBRA 2

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LESSON #3: WRITING AND SOLVING REAL-WORLD EXPONENTIAL GROWTH/DECAY PROBLEMS

Do Now:

A rabbit population doubles every week. There are currently five rabbits in a restricted area. If *t* represents the time, in weeks, and $P(t)$ is the population of rabbits with respect to time, which of the following equations could be used to find how many rabbits will there be in 3 months?

inc by 100% = r

$4 \times 3 = 12$ weeks

1) $P(t)=5(2)^3$
2) $P(t)=5(2)^{12}$
3) $P(t)=(10)^3$
4) $P(t)=(10)^{12}$

$y = P(1+r)^t$

$y = 5(1+1)^{12}$

$y = 5(2)^{12}$

1) a) A rabbit population doubles every 4 weeks. There are currently five rabbits in a restricted area. If *t* represents the time, in weeks, and $P(t)$ is the population of rabbits with respect to time, about how many rabbits will there be in 98 days?

100% ↑ = r → time increment

P = 5

$\frac{98}{7 \text{ days}} = 14$ weeks

$A = P(1+r)^t$

$A = 5(2)^{14/4}$ → Exponent: $\frac{\text{total time } (t)}{\text{time increment } (T)}$

$A = 56.56$

$\boxed{A = 56 \text{ rabbits}}$

b) How many days will it take for the rabbit population to reach 1,000 rabbits? Round answer to *nearest integer*.

↳ A

t → weeks

$\frac{1{,}000}{5} = \frac{5(2)^{\frac{t}{4}}}{5}$

$200 = 2^{\frac{t}{4}}$

$\log_2 200 = \frac{t}{4}$

$\frac{7.6438}{1} = \frac{t}{4}$

$t = 30.5754$

$\times 7$ days

$t = 214.0278$

$\boxed{t = 214 \text{ days}}$

$r = 50\% = .5$ increment P

2) The half-life of a radioactive isotope is 3 days. If there are 40 milligrams of this isotope present, determine, to the *nearest tenth*, the amount of time it will take this isoptope to reduce to approximately 15 milligrams.

A

$A = P(1+r)^{t/T}$

$\frac{15}{40} = \frac{40(1-.5)^{\frac{t}{3}}}{40}$

$.375 = (.5)^{\frac{t}{3}}$

$\log_{.5} .375 = \frac{t}{3}$

$1.4150 = \frac{t}{3}$

$t = 4.2451$

$\boxed{t = 4.2 \text{ days}}$

3) a) The half-life of uranium-232 is 68.9 years. How much of a 100-gram sample is present after 250 years? *Algebraic solution only.* increment (T)

P

$r = .5$ t

$A = P(1 \mp r)^{t/T}$

$A = 100(1-.5)^{\frac{250}{68.9}}$

$\boxed{A = 8.0854 \text{ grams}}$

t P

b) Approximately how many days will it take for the 100-gram uranium sample to breakdown to 1-gram? Round answer to *nearest integer*.

A

$\frac{1}{100} = \frac{100(1-.5)^{\frac{t}{250}}}{100}$

$.01 = (.5)^{\frac{t}{250}}$

$\log_{.5} .01 = \frac{t}{250}$

$\frac{6.6438}{1} = \frac{t}{250}$

$t = 1660.9640$ years

1660.9640

x 365 days/year

$\boxed{606{,}251 \text{ days}}$

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Practice

P r = 1

4) Drew's parents invested $2,500 in an account such that the value of the investment doubles every seven years. How many years, to the *nearest tenth of a year*, will it take the value of the investment to reach $1,000,000?

increment

A

$$A = P(1+r)^{t/T}$$

$$\frac{1000000}{2500} = \frac{2500}{2500}(1+1)^{t/7}$$

$$400 = (2)^{\frac{t}{7}}$$

$$\log_2 400 = \frac{t}{7}$$

$$\frac{8.6438}{1} = \frac{t}{7}$$

$t = 60.5069$

$\boxed{t = 60.5 \text{ years}}$

P

5) a) A substance has a half-life of 6,000 years. If there is 1250 milligrams of the substance present, how much of the substance will remain after 10,000 years? Round to *nearest integer.*

r = .5 increment t

$$A = 1250(1-.5)^{\frac{10,000}{6000}}$$

$$A = 393.7253$$

$\boxed{A = 394 \text{ years}}$

$\frac{1}{5} = .2$

b) After *t* years, one-fifth of the original sample remains radioactive. If there are 1250 milligrams of the sample, solve for *t*, to the *nearest thousand years*.

$$1250 \times .2 = 250 = A$$

$$\frac{250}{1250} = \frac{1250}{1250}(.5)^{\frac{t}{6000}}$$

$$.2 = (.5)^{\frac{t}{6000}}$$

$$\frac{\log_{.5} .2}{1} = \frac{t}{6000}$$

$$\frac{2.3219}{1} = \frac{t}{6000}$$

$\boxed{t = 13931.568 \text{ yrs}}$

$\boxed{t = 14000 \text{ yrs}}$

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MORE PRACTICE!

.5 increment

1) a) Astatine-218 has a half-life of 2 seconds. Find the amount left from a 500 gram sample of astatine-218 after 25 seconds. Round to *nearest hundredth*.

P

t

$$A = 500(.5)^{\frac{25}{2}}$$

$$\boxed{A = .09 \text{ grams}}$$

b) How many seconds will it take, rounded to *nearest thousandth*, for the 500 gram sample to become 100 grams?

P

A

$$\frac{100}{500} = \frac{500(.5)^{\frac{t}{2}}}{500}$$

$$.2 = .5^{\frac{t}{2}}$$

$$\frac{\log_{.5} .2}{1} = \frac{t}{2}$$

$$t = 4.6438 \qquad \boxed{t = 4.644 \text{ seconds}}$$

→200%   → increment

2) The number of bacteria present in a Petri dish triples every 4 hours. Using an exponential model, determine, to the *nearest hundredth*, the number of hours it will take for 50 bacteria to reach 30,700.

P   A

$$\frac{30{,}700}{50} = \frac{50(3)^{\frac{t}{4}}}{50}$$

$$614 = 3^{\frac{t}{4}}$$

$$\frac{\log_3 614}{1} = \frac{t}{4}$$

$$t = 23.3749$$

$$\boxed{t = 23.37 \text{ hrs}}$$