Name: Key

CC GEOMETRY

Date: 5/14/18

TROICI

LESSON #3: ARC LENGTH

Do Now:

1. In the diagram below, tangent ML and secant MNK are drawn to circle O. The ratio $m\overset{\frown}{LN} : m\overset{\frown}{NK} : m\overset{\frown}{KL}$ is $3:4:5$. Find $m\angle LMK$.

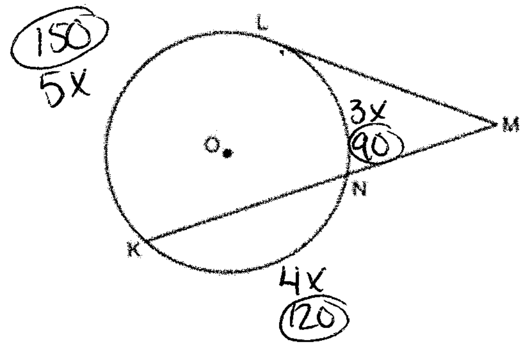
$3x + 4x + 5x = 360$

$12x = 360$

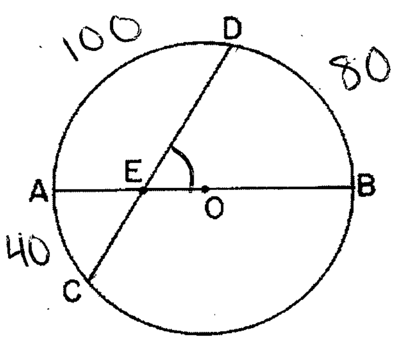
$x = 30$

$\frac{150-90}{2} = \angle LMK$

$\boxed{30° = \angle LMK}$



2. In the accompanying diagram, $\overline{AB}$ is a diameter of circle O and chord $\overline{CD}$ intersects diameter $\overline{AB}$ at E. If $m\overset{\frown}{AD} = 100$ and $m\overset{\frown}{AC} = 40$, find $m\angle DEB$.

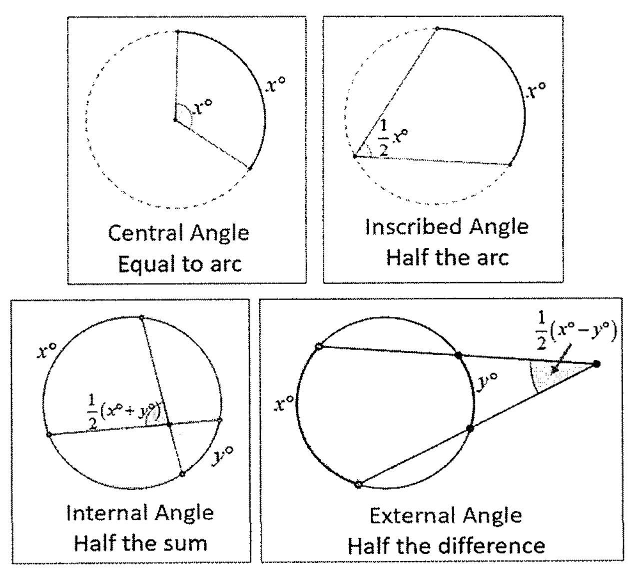


$180 - 100 = 80 = \overset{\frown}{DB}$

$\frac{80+40}{2} = \angle DEB$

$\boxed{60° = \angle DEB}$

Angle-Arc Relationship



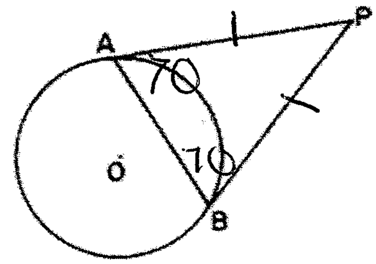
Theorems So Far:

Theorem #1: If two tangents are drawn from the same external point, then the tangents are congruent.

3. **In the accompanying diagram, PA and PB are tangents drawn to circle O. If m$\angle PBA$ = 70, find m$\angle P$.**

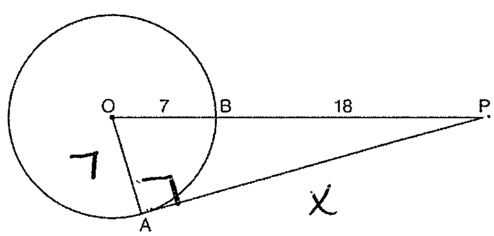
$180 - (70 + 70) = 40$

$\boxed{\angle P = 40^\circ}$



Theorem #2: A tangent and a radius intersect to form a right angle. A tangent and a diameter also intersect to form a right angle.

4. In the diagram below of $\triangle PAO$, $\overline{AP}$ is tangent to circle O at point A, $OB = 7$, and $BP = 18$.



pythagorean thm!

What is the length of $\overline{AP}$?

$7^2 + x^2 = (18+7)^2$

$49 + x^2 = 625$

$-49 \qquad -49$

$\sqrt{x^2} = \sqrt{576}$

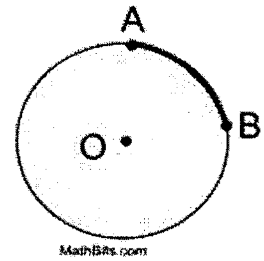
$x = 24$

$\boxed{\overline{AP} = 24}$

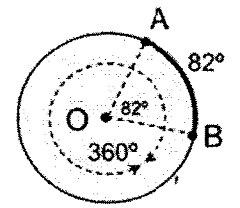
ARC LENGTH!

1) Arc of a circle: is a "portion" of the circumference of the circle.

2) Length of an Arc: is simply the length of its "portion" of the circumference. The circumference itself can be considered a full circle arc length.

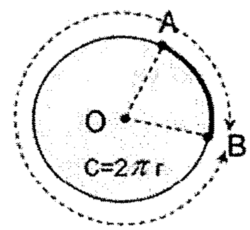


3) Arc measure: In a circle, the degree measure of an arc is equal to the measure of the central angle that intercepts the arc. measured in DEGREES!



arc measure

4) Arc length: In a circle, the length of an arc is a portion of the circumference. The letter "*s*" is used to represent arc length measured in cm, ft, in, mm, etc.

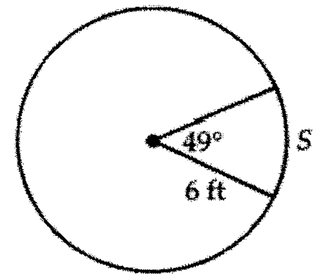


arc length

Formula for finding the <u>arc length</u> in <u>DEGREES</u>:

$$\text{Arc length } (s) = \frac{\text{central } \angle}{360}(2\pi r)$$

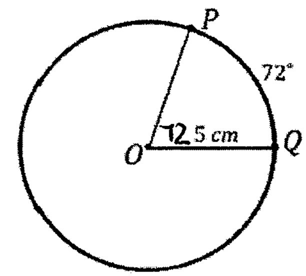
1. What is the arc length in feet? (*round to the nearest hundredth*).



Arc length $= \frac{49}{360}(2\pi(6))$

$\boxed{S = 5.13\text{ft}}$

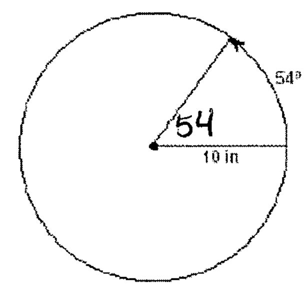
2. *P* and *Q* are points on the circle of radius 5 cm, and the measure of arc $\overset{\frown}{PQ}$ is 72°. Find, *to the nearest tenth*, the length of $\overset{\frown}{PQ}$



Arc length $= \frac{72}{360}(2\pi(5))$

$\boxed{S = 6.3\text{cm}}$

3. A ball is rolling in a circular path that has a radius of 10 inches, as shown in the accompanying diagram. What distance has the ball rolled when the subtended arc is 54°? Express your answer to the *nearest hundredth of an inch.*



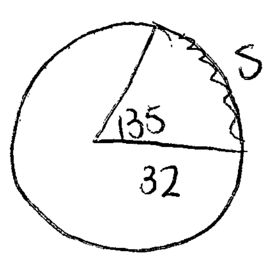
Arc length $= \frac{54}{360}(2\cdot\pi\cdot 10)$

$\boxed{S = 9.42\text{in}}$

4. A brand new circular track was built on the Cougar football field. Josh decided to test it out and took a walk. If the path that Josh followed created a central angle of 135 degrees and the radius of the track is 32 meters, calculate *to the nearest tenth of a meter* how far Josh walked.

$S = \frac{135}{360}(2\pi(32))$

$\boxed{S = 75.4\text{m}}$



x S

5. What is the radius of a circle if the length of a $\underline{45°}$ arc is $\underline{9\pi}$?

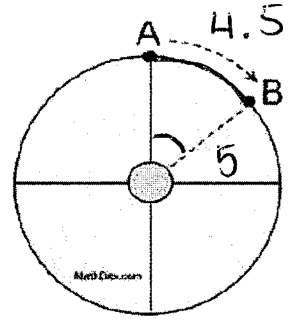
$$S = \frac{x}{360}(2\pi r)$$

$$9\pi = \frac{45}{360}(2\pi r)$$

$$\frac{9\pi}{.25\pi} = \frac{.25\pi r}{\cancel{.25\pi}}$$

$$\boxed{36 = r}$$

6. A child pushes a playground merry-go-round so handle A moves to position B. The radius of the merry-go-round is 5 feet and the distance traveled by the handle along the arc from A to B is 4.5 feet. Find to the *nearest* $\underline{\textit{degree}}$, the measure of minor arc



$$S = \frac{x}{360}(2\pi \cdot 5)$$

$$4.5 = \frac{x}{360}(2\pi \cdot 5)$$

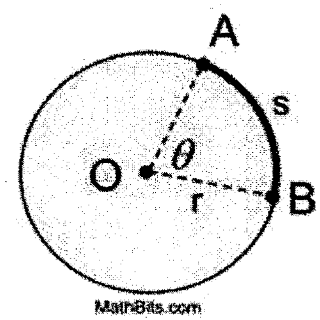
$$4.5 = \frac{10\pi x}{360}$$

$$\frac{4.5}{.0872} = \frac{.0872x}{.0872}$$

$$x = 51.5662$$

$$\boxed{x = 52°}$$

The radian measure, θ, of a central angle is defined as the ratio of the length of the arc the angle subtends, s, divided by the radius of the circle, r.

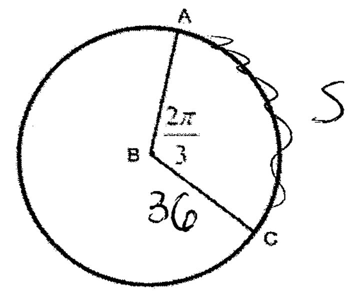


Formula for finding the <u>arc length</u> in <u>RADIANS</u>:

Arclength = central ∠ · Radius

$\boxed{S = \theta r}$

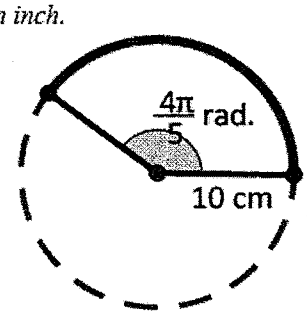
7. The radius of the following circle is 36 cm. What is the arc length of $\overset{\frown}{AC}$? Keep your answer in terms of π.



$S = \frac{2\pi}{3} \cdot 36$

$\boxed{S = 24\pi \text{ cm}}$ ~~unidades~~

8. Determine the arc length with the central angle given as radians, *nearest tenth of an inch.*



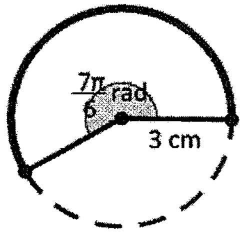
$S = \theta r$

$S = \frac{4\pi}{5} \cdot 10$

$\boxed{S = 25.1 \text{ in}}$

9. Determine the arc length of the following examples with the central angle given as radians.

(a)



$s = \theta r$

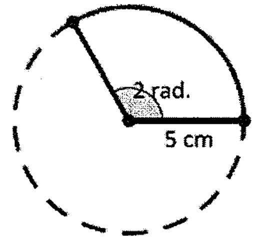
$s = \frac{7\pi}{6} \cdot 3$

$s = 3.5\pi$

$s = 10.9955$

$\boxed{s = 11 cm}$

(b)

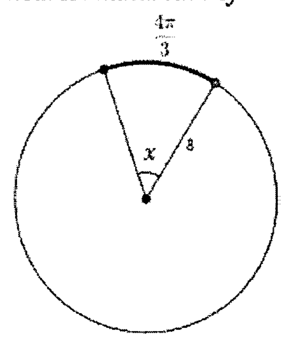


$s = \theta r$

$s = 2 \cdot 5$

$\boxed{s = 10 cm}$

10. In the circles shown, find the value of x. Figures are not drawn to scale, round your answer to *nearest hundredth of an inch.*



*

$s = \theta r$

$\frac{\frac{4\pi}{3}}{8} = \frac{\theta \cdot 8}{8}$

$\boxed{\theta = .52 in}$

~~$s = \frac{\theta}{360} 2\pi r$~~

~~$\frac{4\pi}{3} = \frac{\theta}{360} 2\pi 8$~~

~~$\frac{\frac{4\pi}{3}}{.04} = \frac{.04\theta}{.04}$~~

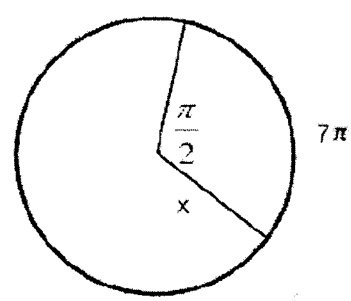
~~$\theta = 30.00$~~

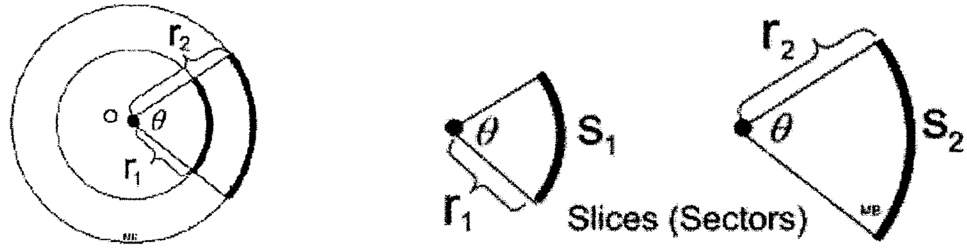
11. In the circles shown, find the value of x. Figures are not drawn to scale.

$\frac{7\pi}{1} = \frac{\pi}{2} \cdot x$

$14\cancel{\pi} = \cancel{\pi} x$

$\boxed{x = 14}$





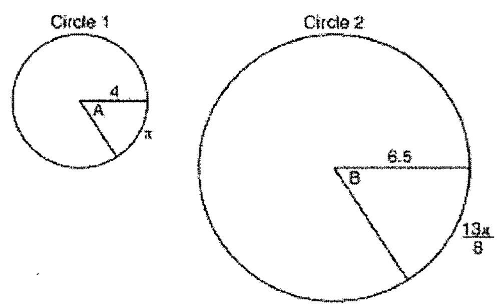
The same dilation that mapped the smaller circle onto the larger circle will also map the slice (sector) of the smaller circle with an arc length of s_1 onto the slice (sector) of the larger circle with an arc length of s_2. If the radius gets dilated by a scale factor, then arc length is also dilated by that same scale factor. Since corresponding parts of similar figures are in proportion, $\frac{r_1}{r_2} = \frac{s_1}{s_2}$.

12. In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle A intercepts an arc of length π, and angle B intercepts an arc of length $\frac{13\pi}{8}$.

$s = \theta r$

$\frac{\pi}{4} = \frac{4\theta}{4}$

$\theta = \frac{\pi}{4}$ ✓



$s = \theta r$

$\frac{\frac{13\pi}{8}}{6.5} = \frac{6.5\theta}{6.5}$

$\theta = \frac{\pi}{4}$ ✓

Dominic thinks that angles A and B have the same radian measure. State whether Dominic is correct or not. Explain why.

① All circles are ~, corresponding parts are proportion

$\frac{r_1}{r_2} = \frac{s_1}{s_2} \Rightarrow \frac{4}{6.5} = \frac{\pi}{\frac{13\pi}{8}}$

$\frac{13}{2}\pi = 6.5\pi$

$6.5\pi = 6.5\pi$ ✓

EXTRA PRACTICE!!!

S r

1) An arc has a length of 7 kilometers in a circle with a radius of 12 kilometers. What is the measure of the arc's central angle, *to the nearest degree*?

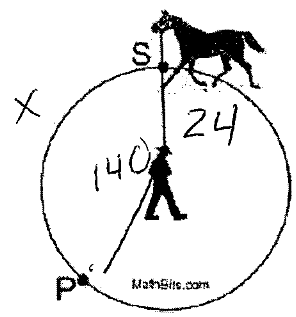
$$s = \frac{\theta}{360}(2\pi r)$$

$$7 = \frac{\theta}{360}(2\pi \cdot 12)$$

$$7 = .2094\theta$$

$$\boxed{\theta = 33^\circ}$$

2) A horse on a lunge line travels in a circle around its trainer. The radius of the horse's circle is 24 feet. If the angle between location *S* and *P* on the circle is 140° (counterclockwise), find the length the horse travels from *S* to *P* in feet, to the *nearest foot*.

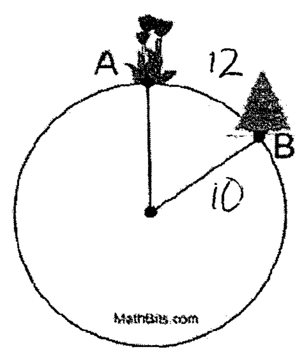


$$s = \frac{140}{360}(2\pi \cdot 24)$$

$$s = 58.6430$$

$$\boxed{s = 59\text{ ft}}$$

3) A sprinkler head is equidistant from flower garden *A* and a small shrub *B*. The sprinkler waters in a circular pattern. If the length of minor arc *AB* is 12 feet and the radius of the circle is 10 feet, find the measure of the central angle subtended by minor arc *AB*, to the *nearest degree.*



$$s = \frac{\theta}{360}(2\pi r)$$

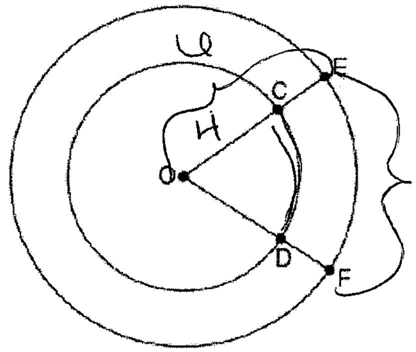
$$12 = \frac{\theta}{360}(2\pi 10)$$

$$12 = .1745\theta$$

$$\theta = 68.7549$$

$$\boxed{\theta = 69^\circ}$$

4) In the diagram below, two concentric circles with center O, and radii $\overline{OC}$, $\overline{OD}$, $\overline{OGE}$, and $\overline{ODF}$ are drawn.



If $OC = 4$ and $OE = 6$, which relationship between the length of arc EF and the length of arc CD is always true?

1) The length of arc EF is 2 units longer than the length of arc CD.

2) The length of arc EF is 4 units longer than the length of arc CD.

3) The length of arc EF is 1.5 times the length of arc CD.

~~4)~~ The length of arc EF is 2.0 times the length of arc CD.

$EF : CD$

$6 : 4 = \frac{6}{4} = 1.5$