

LESSON #2: QUESTIONS WITH CHALLENGING EXPONENTS

Do Now:

1. The value of an initial investment of \$400 at 3% nominal interest compounded quarterly can be modeled using which of the following equations, where t is the number of years since the investment was made?

- (1) $A = 400(1.0075)^{4t}$ (3) $A = 400(1.03)^{4t}$ $A = P(1 + \frac{r}{n})^{nt}$
 (2) $A = 400(1.0075)^t$ (4) $A = 400(1.0303)^{4t}$ $A = 400(1 + \frac{.03}{4})^{4t}$
 $A = 400(1.0075)^{4t}$

2. True or false?

$5^{\frac{x}{12}} = \left(5^{\frac{1}{12}}\right)^x$ True!
 multiply $\frac{1}{12} \cdot \frac{x}{1} = \frac{x}{12}$

3. Rewrite $3^{\frac{x}{10}}$ as a number raised to the x power.

$3^{\frac{x}{10}} = (3^{\frac{1}{10}})^x$

1) A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, $B(t)$, can be represented by the function $B(t) = 750(1.16)^t$, where the t represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

- a. $B(t) = 750(1.012)^t$
 (b) $B(t) = 750(1.012)^{12t}$
 c. $B(t) = 750(1.16)^{12t}$
 d. $B(t) = 750(1.16)^{\frac{1}{12}}$

$750(1.16)^t = 750(1.16^{\frac{1}{12}})^{12t}$ why?
 $750(1.0124)^{12t}$ $\frac{1}{12} \cdot 12t = t$ equal!

2) A payday loan company makes loans between \$100 and \$1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a \$300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

2 weeks
 ~ 26 times
 per year

(1) $300(.30)^{\frac{14}{365}}$ decay
 (2) $300(1.30)^{\frac{14}{365}}$
 ↓
 .038

(3) $300(.30)^{\frac{365}{14}}$ decay
 (4) $300(1.30)^{\frac{365}{14}}$
 ↓
 26.07 ✓

$A = 300(1 + .30)^{\frac{365}{14}}$

~ 26 times a year

3) One of the medical uses of Iodine-131 (I-131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I-131 is approximately 8.02 days. Decay!

a. Which equation represents this situation?

~~(1)~~ $A = P(1.5)^{\frac{t}{8.02}}$

(3) $A = P\left(\frac{1}{2}\right)^{\frac{t}{8.02}}$

(2) $A = P\left(\frac{1}{2}\right)^{\frac{t}{8.02}}$

~~(4)~~ $A = P(2)^{\frac{t}{8.02}}$

$A = P(1-r)^t$

$A = P(1-.5)^t$

THINK! what if it was every 10 days over a 50 day period

How many times? 5! why? $\frac{50}{10} = 5$

b. A patient is injected with 20 milligrams of I-131. Using the equation from part a, determine, to the nearest day, the amount of time needed before the amount of I-131 in the patient's body is approximately 7 milligrams.

$A = 7$

$P = 20$

$t = ?$

initial

$A = P\left(\frac{1}{2}\right)^{\frac{t}{8.02}}$

$7 = 20\left(\frac{1}{2}\right)^{\frac{t}{8.02}}$

$\frac{7}{20} = \left(\frac{1}{2}\right)^{\frac{t}{8.02}}$

LOG IT! $.35 = \left(\frac{1}{2}\right)^{\frac{t}{8.02}}$

$\hookrightarrow \log .35 = \frac{t}{8.02} \log \frac{1}{2}$

$\frac{.5145}{.301} = \frac{t}{8.02}$

$t = 12.1468$

$t = 13 \text{ days}$

4) Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [*Let m represent months.] →

GROWTH!

(1) $(1.0525)^m$

(3) $(1.00427)^m$

(2) $(1.0525)^{\frac{12}{m}}$

(4) $(1.00427)^{\frac{m}{12}}$

$\left(\left(1 + .0525\right)^{\frac{1}{12}}\right)^m$
 $(1.0042)^m$

5) An equation to represent the value of a car after t months of ownership is

$v = 32,000(0.81)^{\frac{t}{12}}$. Which statement is **not** correct?

(1) The car lost approximately 19% of its value each month.

~~(2)~~ The car maintained approximately 98% of its value each month.

~~(3)~~ The value of the car when it was purchased was \$32,000. $A = 32,000$

~~(4)~~ The value of the car 1 year after it was purchased was \$25,920.

SUB $t = 12$ and check in calc

$32000 (.81^{\frac{1}{12}})^t$

$(.9825...)$

$1 - r = .9825$

$-1 \quad -1$

$-r = -.0174$

$r = .0174$

$1.7 \approx 2\% \text{ decay}$

LAB #2

- 1) Kristen invests \$5,000 in a bank. The bank pays 6% interest compounded monthly. To the nearest tenth of a year, how long must she leave the money in the bank for it to double?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$\frac{10,000}{5000} = \frac{5,000}{5000} \left(1 + \frac{.06}{12}\right)^{12t}$$

$$2 = (1.005)^{12t}$$

$$\frac{\log_{1.005} 2}{12} = \frac{12t}{12}$$

$$t = 11.58$$

$t = 11.6 \text{ years}$

- * 2) Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium-44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time.

decay!

- ~~(1)~~ $A(t) = 100(1 + .5)^{\frac{t}{63}}$ **(3)** $A(t) = 100(1 - .5)^{\frac{t}{63}}$
~~(2)~~ $A(t) = 63(1 - .5)^{\frac{t}{100}}$ ~~(4)~~ $A(t) = 100(1 - .5)^{\frac{63}{t}}$

- 3) Last year, the total revenue for Outback Steakhouse, increased 7% over the previous year. If this trend were to continue, which expression could the company's CFO use to approximate their monthly percent increase in revenue? [*Let m represent months.]

growth!

- (1) $(1.07)^m$ **(3)** $(1.00565)^m$ $(1 + .07 \frac{1}{12})^m$
 (2) $(1.07)^{\frac{12}{m}}$ (4) $(1.00565)^{\frac{m}{12}}$ $(1.00565)^m$

