

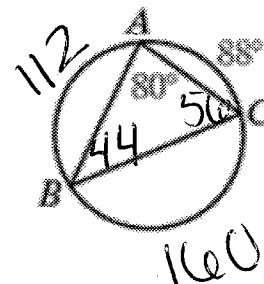
LESSON #2: INTERIOR AND EXTERIOR ANGLES

Do Now:

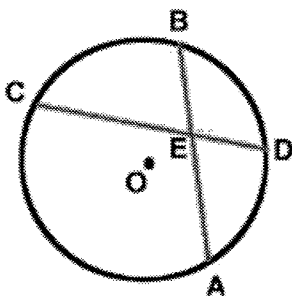
- A central angle is equal to its intercepted arc.
- An inscribed angle is Half of its intercepted arc.

Triangle ABC is inscribed in a circle, $m\angle A = 80$ and $m\widehat{AC} = 88$. Find:

- a. $m\widehat{BC}$ b. $m\angle B$ c. $m\angle C$ d. $m\widehat{AB}$ e. $m\widehat{BAC}$
- 160° 44 56 112 200



HOW DO WE FIND THE MEASURE OF INTERIOR ANGLES OF A CIRCLE?!?!

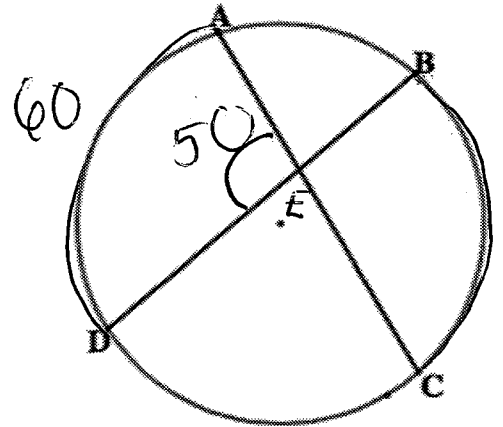


$$m\angle BED = \frac{1}{2}(m\widehat{BD} + m\widehat{CA})$$

$$m\angle DEA = \frac{1}{2}(m\widehat{DA} + m\widehat{CB})$$

1. In the accompanying diagram of circle O , if $m\widehat{CB} = 40^\circ$ and $m\widehat{AD} = 60^\circ$. What is the value of $m\angle AED$?

$$m\angle AED = \frac{60 + 40}{2} = \boxed{50^\circ}$$



2. In circle O , chords \overline{AB} and \overline{CD} intersect at E . If $m\angle AED = 75^\circ$ and $m\widehat{CB} = 40^\circ$. What is the value of \widehat{AD} ?

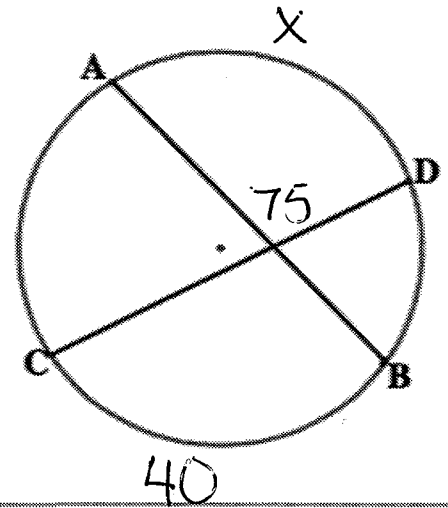
$$\frac{\text{arc} + \text{arc}}{2} = \angle$$

$$\frac{40 + x}{2} = \frac{75}{1}$$

$$\begin{array}{r} 40 + x = 150 \\ -40 \quad -40 \\ \hline \end{array}$$

$$\boxed{x = 110^\circ}$$

$$\boxed{m\widehat{AD} = 110^\circ}$$

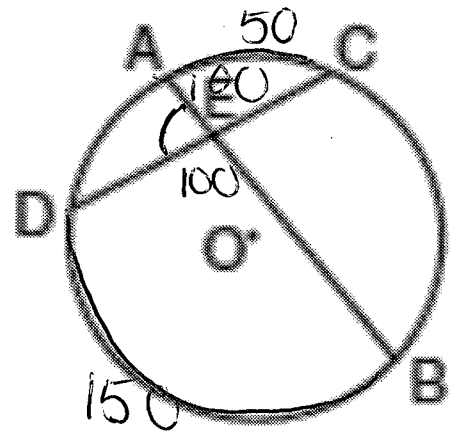


3. In the accompanying diagram of circle O , chords \overline{AB} and \overline{CD} intersect at E , $m\widehat{AC} = 50$, and $m\widehat{BD} = 150$. Find $m\angle AED$.

$$\frac{50 + 150}{2} = \frac{200}{2} = 100$$

$$180 - 100 = 80^\circ$$

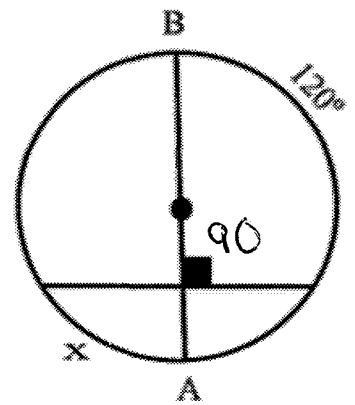
$$\boxed{m\angle AED = 80^\circ}$$



4. Given diameter \overline{AB} perpendicular to a chord, find x .

$$\frac{120+x}{2} = \frac{90}{1}$$

$$\begin{array}{r} 180 = 120 + x \\ -120 \quad -120 \\ \hline \boxed{x = 60^\circ} \end{array}$$

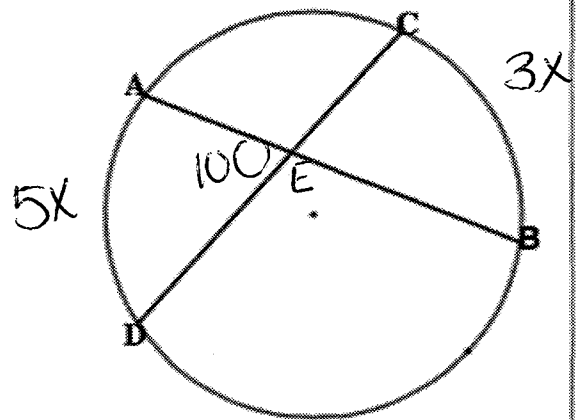


5. In the accompanying diagram of circle O , if $m\angle AED = 100^\circ$ and $m\widehat{CB} : m\widehat{AD} = 3:5$. What is the value of $m\widehat{CB}$?

$$\frac{5x+3x}{2} = \frac{100}{1}$$

$$\begin{array}{l} 8x = 200 \\ x = 25 \end{array}$$

$$m\widehat{CB} = 3(25) = \boxed{75^\circ}$$



6. Two chords intersect within a circle to form an angle whose measure is 53° . If the intercepted arcs are represented by $3x + 3$ and $10x - 14$, find the measure of larger of these two arcs.

$$\frac{3x+3+10x-14}{2} = \frac{53}{1}$$

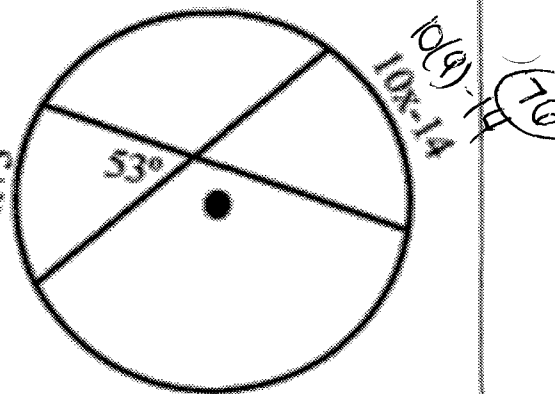
$$\begin{array}{r} 106 = 13x - 11 \\ + 11 \qquad \qquad + 11 \\ \hline 117 = 13x \end{array}$$

$$117 = 13x$$

$$x = 9$$

$$76^\circ$$

$$\begin{array}{r} 30 \\ 3(9) \overline{) 30} \\ \underline{27} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

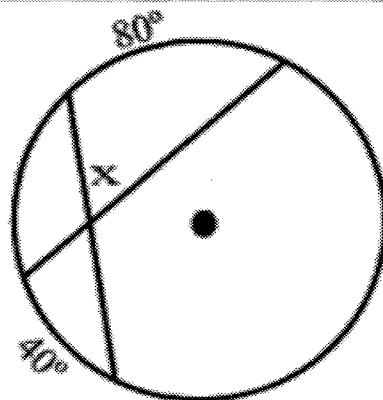


7. Given circle to the right, find x .

$$\frac{80+40}{2} = \frac{x}{1}$$

$$120 = 2x$$

$$x = 60^\circ$$

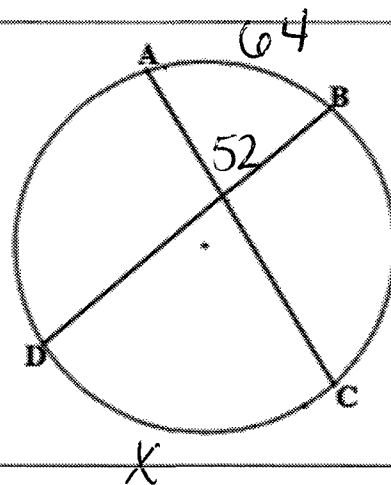


8. Given circle to the right, In circle O , chords \overline{AB} and \overline{CD} intersect at E . If $m\angle AEB = 52^\circ$ and $AB = 64$. What is the value of \widehat{CD} ?

$$\frac{64+x}{2} = \frac{52}{1}$$

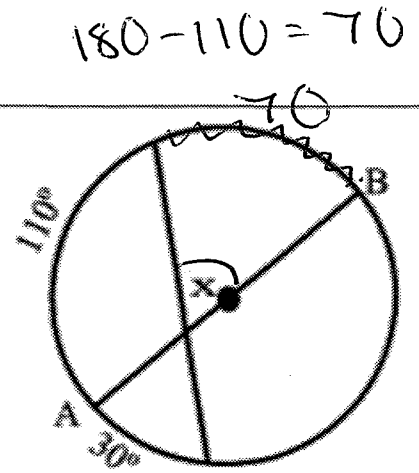
$$64+x = 104$$

$$x = 40$$



9. Given circle to the right, diameter \overline{AB} and chord, find x .

$$\frac{70+30}{2} = \frac{100}{2} = \boxed{50^\circ = x}$$



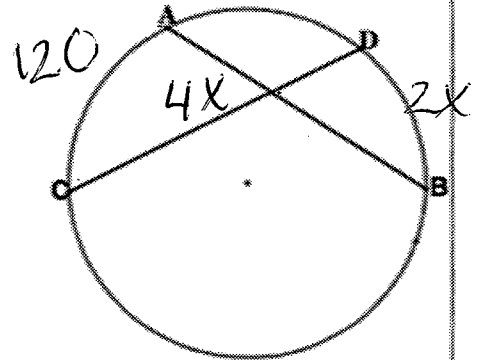
10. In circle O , chords \overline{AB} and \overline{CD} intersect at E . If $m\angle AEC = 4x$ and $AC = 120^\circ$ and $DB = 2x$. What is the value of x ?

$$\frac{120 + 2x}{2} = \frac{4x}{1}$$

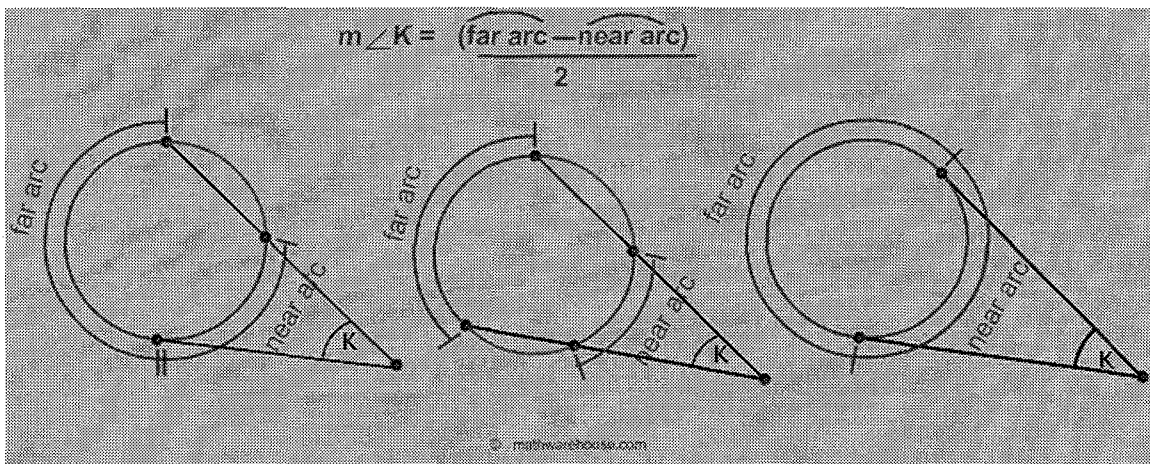
$$8x = 120 + 2x$$

$$6x = 120$$

$$\boxed{x = 20}$$

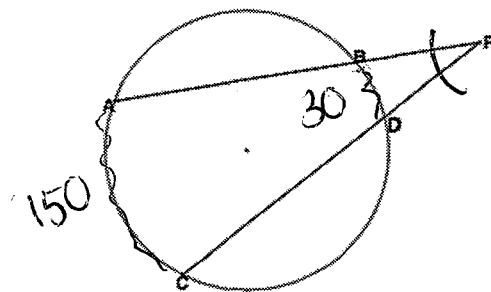


HOW DO WE FIND THE MEASURE OF EXTERIOR ANGLES OF A CIRCLE?!?!



1. If $m\widehat{AC} = 150^\circ$ and $m\widehat{BD} = 30^\circ$. What is $m\angle P$?

$$\frac{150 - 30}{2} = \frac{120}{2} = \boxed{60^\circ}$$



2. If $m\angle P = 30^\circ$ and $m\widehat{AC} = 200^\circ$. What is $m\widehat{BD}$?

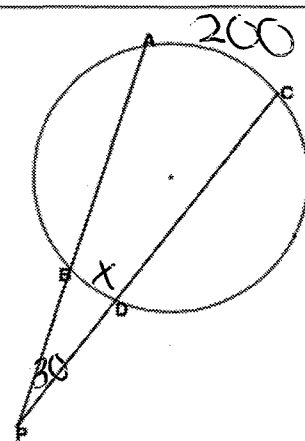
$$\frac{200 - x}{2} = \frac{30}{1}$$

$$200 - x = 60$$

$$\begin{array}{r} -200 \quad -200 \\ 200 - x = 60 \\ -200 \quad -200 \end{array}$$

$$-x = -140$$

$$\boxed{x = 140^\circ}$$



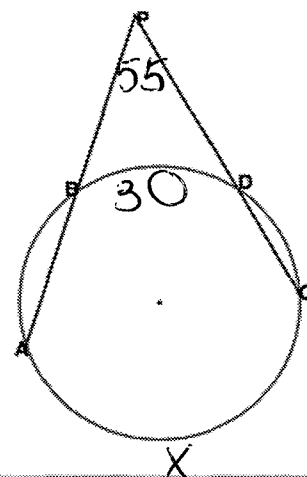
3. If $m\angle P = 55^\circ$ and $m\widehat{BD} = 30^\circ$. What is $m\widehat{AC}$?

$$\frac{x - 30}{2} = \frac{55}{1}$$

$$110 = x - 30$$

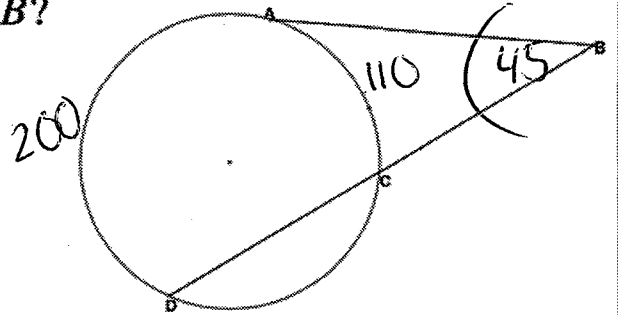
$$\begin{array}{r} +30 \quad +30 \\ 110 = x - 30 \\ +30 \quad +30 \end{array}$$

$$\boxed{x = 140^\circ}$$



4. If $m\angle AC = 110^\circ$ and $m\angle AD = 200^\circ$. What is $m\angle B$?

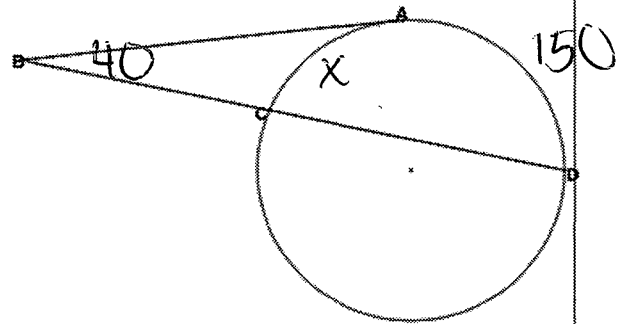
$$\frac{200 - 110}{2} = \boxed{45^\circ}$$



5. If $m\angle B = 40^\circ$ and $m\angle AD = 150^\circ$. What is $m\angle AC$?

$$\begin{aligned} \frac{150 - x}{2} &= \frac{40}{1} \\ 80 &= 150 - x \\ -150 &-150 \\ \hline -70 &= -x \end{aligned}$$

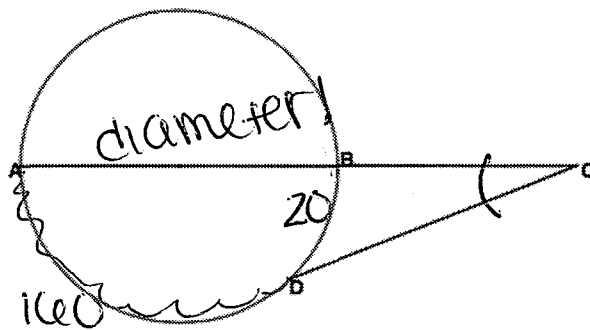
$$\boxed{x = 70^\circ}$$



6. If $m\widehat{BD} = 20^\circ$. What is $m\angle C$?

$$\frac{160 - 20}{2} = x$$

$$\boxed{70 = x^\circ}$$



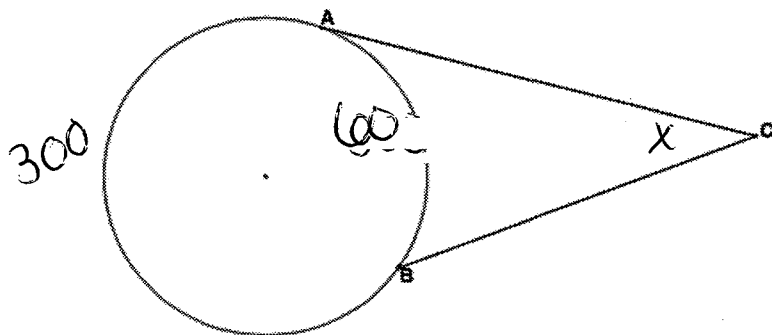
$$180 - 20 = 160$$

7. If $m\widehat{AB} = 300^\circ$. What $m\angle C$?

$$\frac{300 - 60}{2} = x$$

$$\boxed{120^\circ = x}$$

$$360 - 300 = 60$$

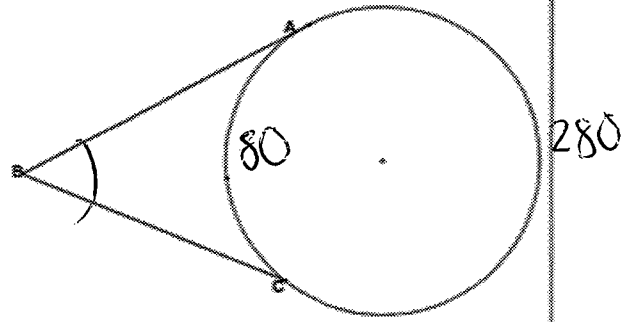


8. If $m\widehat{AC} = 80^\circ$. What $\angle B$?

$$360 - 80 = 280$$

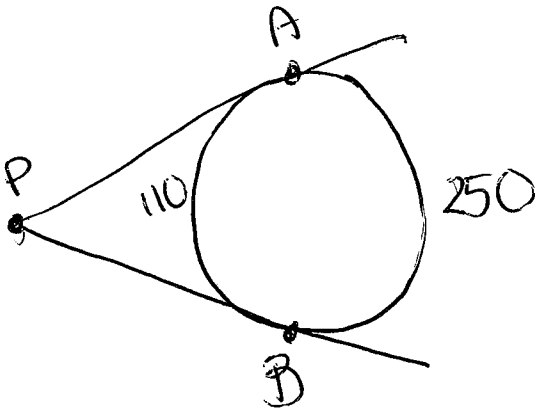
$$\frac{280 - 80}{2} = x$$

$$\boxed{100 = x}$$



9. Two tangents \overline{PA} and \overline{PB} are drawn to circle O from an external point P . If the measure of major arc \widehat{AB} is 250° , find the measure of $\angle P$.

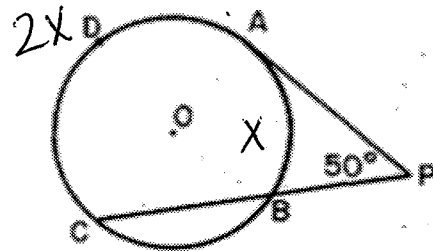
$$\frac{250 - 110}{2} = \boxed{70^\circ}$$



10. In the accompanying diagram, tangent \overline{PA} and secant \overline{PBC} are drawn to circle O . If $m\widehat{ADC}$ is twice $m\widehat{AB}$ and $m\angle P$ is 50, what is $m\widehat{AB}$?

$$\frac{2x - x}{2} = \frac{50}{1}$$

$$\boxed{100^\circ = x}$$

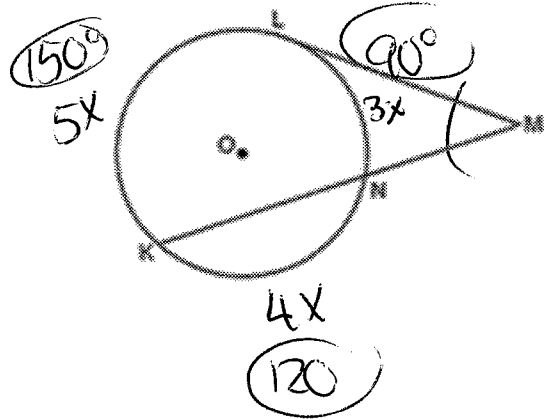


11. In the diagram below, tangent \overline{ML} and secant \overline{MNK} are drawn to circle O . The ratio $m\widehat{LN} : m\widehat{NK} : m\widehat{KL}$ is 3:4:5. Find $m\angle LMK$.

$$3x + 4x + 5x = \frac{12x}{12} = \frac{360}{12}$$

$$x = 30$$

$$\frac{150 - 90}{2} = \boxed{30^\circ}$$

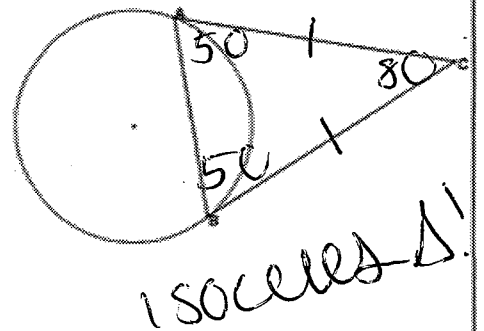


THEOREM: If two tangents are drawn from the same external point, then the tangents are \cong

12. If $m\angle C = 80^\circ$, then what is $m\angle CAB$?

$$180 - 80 \div 2 = 50$$

$$\boxed{m\angle CAB = 50^\circ}$$

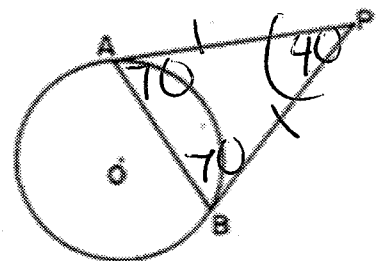


13. In the accompanying diagram, \overline{PA} and \overline{PB} are tangents drawn to circle O . If $m\angle PBA = 70$, find $m\angle P$.

$$70 + 70 = 140$$

$$180 - 140 = 40$$

$$\boxed{m\angle P = 40^\circ}$$

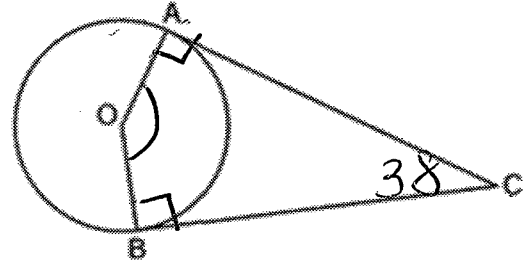


THEOREM: A tangent & a radius intersect & form right \angle 's. A tangent & a diameter intersect to form right \angle 's

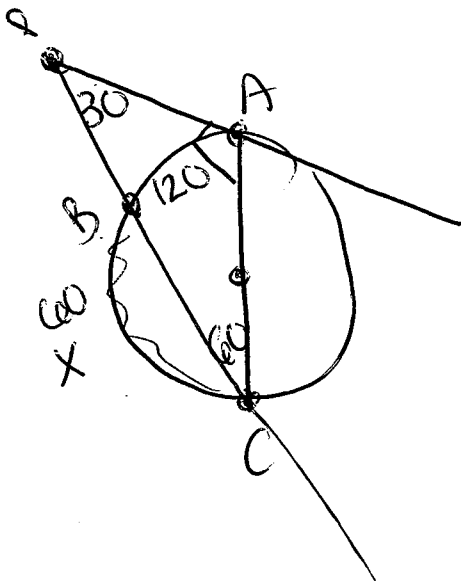
14. In the diagram below, \overline{AC} and \overline{BC} are tangent to circle O at A and B , respectively, from external point C . If $m\angle ACB = 38$, what is $m\angle AOB$?

work $90 + 90 + 38 = 218$
 $360 - 218 = 142^\circ$

$m\angle AOB = 142^\circ$



15. Point P lies outside circle O , which has a diameter of \overline{AC} . The angle formed by tangent \overline{PA} and secant \overline{PBC} measures 30° . Sketch the conditions given above and find the number of degrees in the measure of minor arc CB .



$30 + 90 = 120$
 $180 - 120 = 60$

$x = 60^\circ$

Xtra help:

Nicole - Test

Dom - Test

Vee - Notes