

LESSON #1: LOG WORD PROBLEMS

Do Now:

1. A loan for \$8,000 compounds yearly at a rate of 2.5%. What will the cost of the loan after 3 years?  
 (HINT: Use the formula  $A = P(1+r)^t$ )

$A = ?$        $t = 3$        $A = 8000(1 + .025)^3 = \boxed{\$8615.13}$   
 $P = 8000$   
 $r = .025$

2. A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the nearest percent. (HINT: Use the formula  $A = P(1+r)^t$ )

$A = 135,000$        $\frac{135,000}{100,000} = \frac{100,000(1+r)^5}{100,000}$        $1.0618 = 1+r$   
 $P = 100,000$        $(1.35)^{1/5} = (1+r)^5$        $r = .0618$   
 $r = ?$        $t = 5$        $\text{OR } \approx 6\%$

Compounded Annually	Compounded Semi-annually/ quarterly/ weekly/monthly/daily	Compounded Continuously
$A = P(1+r)^t$ $A = \text{Final amount}$ $P = \text{Initial amount}$ $r = \text{rate as a decimal}$	$A = P(1 + \frac{r}{n})^{nt}$ $A = \text{Final amount}$ $P = \text{Initial}$ $r = \text{rate as decimal}$ $n = \text{compounded}$ $t = \text{time}$	$A = Pe^{rt}$ $A = \text{Final}$ $P = \text{Initial}$ $r = \text{rate as a decimal}$ $t = \text{time}$

SOLVING FOR AN EXPONENT? USE LOGS!!!!!!!!!!!!

1. Find the time, to the nearest year, required for a \$7500 investment to double at an interest rate of 7.25% compounded annually.

$A = 15000$        $A = P(1+r)^t$        $\rightarrow \text{growth} \rightarrow +$   
 $P = 7500$        $15000 = 7500(1 + .0725)^t$   
 $r = .0725$        $\frac{15000}{7500} = \frac{7500(1 + .0725)^t}{7500}$   
 $t = ?$        $2 = (1.0725)^t$        $t = 10 \text{ years}$   
 convert to log!  $\log 2 = \log(1.0725)^t$   
 $t = \log_{1.0725} 2$   
 $t = 9.9031$

2. Going away on a family vacation of four to Disney World is very expensive. The cost of the trip is \$5,500. If you put \$1,000 into an account at a .5% annual interest rate compounded monthly, how long, to the nearest tenth, will it take you to save up for this trip?

$A = 5500$        $A = P(1 + \frac{r}{n})^{nt}$        $n = 12$   
 $P = 1000$        $5500 = 1000(1 + \frac{.005}{12})^{12t}$        $t$   
 $r = .005$        $\frac{5500}{1000} = \frac{1000(1 + \frac{.005}{12})^{12t}}{1000}$        $\rightarrow \text{calcul.}$   
 $n = 12$        $5.5 = (1.0004)^{12t}$   
 $t = ?$        $5.5 = (1.0004)^{12t}$        $12t = \log_{1.0004} 5.5$   
 $12t = 4092.2444$        $t = 341.0 \text{ yrs}$

3. Grace invests \$6,000 in a CD at an annual rate of 8% compounded continuously.  $A = Pe^{rt}$

a) Determine, to the nearest dollar, the amount of money she will have after 5 years.  $\rightarrow t$

$A = 6000 e^{(0.08)(5)} \rightarrow \text{calc}$   
 $A = \$8951$

b) Determine how many years, to the nearest year, it will take for her investment to triple.

$18000 = 6000 e^{.08t}$   $\rightarrow A = 6000 \times 3$   
 $\frac{18000}{6000} = \frac{6000}{6000} e^{.08t}$   $A = 18000$   
 $3 = e^{.08t}$   
 $\ln 3 = \ln e^{.08t}$   
 $\ln 3 = .08t$   
 $t = \frac{\ln 3}{.08} = 13.73 \text{ yrs}$   
 $t = 14 \text{ years}$

Partner Practice:

4. Depreciation of a car's value can be determined by the formula  $V = C(1-r)^t$ , where V is the value of the car after t years, C is the original cost, and r is the rate of depreciation. If a car's cost, when new, is \$15,000, the rate of depreciation is 30%, and the value of the car now is \$3,000. How old is the car to the nearest tenth of a year?

$3000 = 15000(1-.30)^t$   
 $\frac{3000}{15000} = \frac{15000}{15000} (1-.30)^t$   
 $.2 = (.7)^t$   
 $t = \log_{.7} .2 \rightarrow t = 4.5 \text{ years}$

5. Sabrina is saving for a new car which will cost \$15,000. If she puts \$5,000 in an account which earns a 10% interest compounded semi-annually, how long will it take her to save enough money to buy the car? Round to the nearest year.

$A = P(1 + \frac{r}{n})^{nt}$   
 $15000 = 5000(1 + \frac{.10}{2})^{2t}$   
 $\frac{15000}{5000} = \frac{5000}{5000} (1 + \frac{.10}{2})^{2t}$   
 $3 = (1.05)^{2t}$   
 $2t = \log_{1.05} 3$   
 $\frac{2t}{2} = \frac{22.5170}{2}$   
 $t = 11.2585$   
 $t = 12 \text{ years}$

6. Luke invests \$10,000 at an annual rate of 5% compounded continuously.  $A = Pe^{rt}$

a) Determine, to the nearest dollar, the amount of money he will have after 2 years.

$A = 10,000 e^{(0.05)(2)}$   
 $A = 11051.70918$   
 $A = \$11,052$

b) Determine how many years, to the nearest year, it will take for his investment to double.

$20,000 = 10000 e^{.05t}$   $A = 20,000$   
 $\frac{20,000}{10,000} = \frac{10,000}{10,000} e^{.05t}$   
 $2 = e^{.05t}$   
 $\ln 2 = \ln e^{.05t}$   
 $\ln 2 = .05t$   
 $t = \frac{\ln 2}{.05} = 13.8629$   
 $t = 14 \text{ years}$