MRS. WEINSTEIN AND MS. TROICI'S STUDY GUIDE FOR THE GEOMETRY REGENTS EXAM

Regents Exam: June 19th, 2018
9:15 AM

DATES AND TIMES THAT *MRS. WEINSTEIN* IS AVAILABLE FOR EXTRA HELP/STUDY SESSIONS

- Friday, June 8th: Available 12:15 PM – 3:30 PM
- Monday, June 11th: Available ALL DAY
- Tuesday, June 12th: Available 9:00 AM – 12:00 PM
- Wednesday, June 13th: **NOT AVAILABLE** (Grading all day)
- Thursday, June 14th: Available 9:00 AM – 12:00 PM
- Friday, June 15th: **NOT AVAILABLE** (Grading all day)
- Monday, June 18th: Available ALL DAY
- Tuesday, June 19th: **REGENTS EXAM IN THE MORNING**

DATES AND TIMES THAT *MS.TROICI* IS AVAILABLE FOR EXTRA HELP/STUDY SESSIONS

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DON'T FORGET TO GO TO YOUTUBE TO WATCH THE CONSTRUCTION VIDEOS!!

YOUTUBE CHANNEL: WeinsteinMAPMATH

YOU MUST GO TO:

WWW.troicitime.weebly.com

FOR QUIZLET FLASHCARDS AND REGENT EXAMS ANSWER KEYS!

FORMULAS AND IMPORTANT INFORMATION YOU NEED TO KNOW THAT *ARE NOT* ON YOUR REFERENCE SHEET!

1) **Distance Formula:** $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ when given points (x_1, y_1) and (x_2, y_2).

2) **Slope Formula:** $m = \frac{y_2 - y_1}{x_2 - x_1}$ when given points (x_1, y_1) and (x_2, y_2).

3) **Arc Length** = $\left(\frac{x}{360}\right) 2\pi r$ where x stands for the degree measure of the arc, and r stands for the radius of the circle.

4) **Area of a Sector** = $\left(\frac{x}{360}\right) \pi r^2$ where x stands for the degree measure of the arc, and r stands for the radius of the circle.

5) **Density:** $D = \frac{M}{V}$ (the most miserable place on earth!) $Density = \frac{Mass}{Volume}$

6) **General Form of a Circle:** $(x - h)^2 + (y - k)^2 = r^2$ where $(h, k) = center\ of\ the\ circle$ and $r = radius$.

7) **General Form of a Line:** $y = mx + b$ where $m = slope$ and $b = y - intercept$.

8) **Area of a rectangle:** $A = l \times w$

9) **Area of a square:** $A = l \times w$

10) **Volume of a rectangular prism (a box):** $V = l \times w \times h$

11) **Parallel Lines:** Have *equal* slopes.

12) **Perpendicular Lines:** Have *negative reciprocal* slopes.

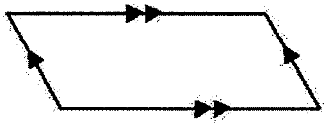
13) $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$: Used only with *right triangles*.

TIPS AND TRICKS FOR THE REGENTS EXAM!!

- When you see the words *angle of elevation* or *angle of depression,* that means the question is going to involve $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$!!
- All triangles add up to 180°.
- All circles add up to 360°.
- Very often, when you see the word *length* that means the problem is going to involve the distance formula!
- When you see the words *right rectangular prism,* it is just a fancy way of saying A BOX!
- When a question refers to *filling something* or the amount of *contents something can hold,* that means you are going to be finding VOLUME!
- When we dilate a figure, the *angle measures* do not change!
- When talking about *parallel lines,* the "Z" trick refers to *alternate interior angles.* The "F" trick refers to *corresponding angles.* Alternate interior angles are congruent. Corresponding angles are congruent.
- When we dilate a line about the *origin* by a scale factor *r,* the slope remains the same. The *y*-intercept gets multiplied by the scale factor.
- The *radius* of a circle is equal to half of the *diameter.*
- An *isosceles triangle* has two congruent sides and two congruent base angles.
- When a *quadrilateral* is *inscribed in a circle,* the opposite angles of the quadrilateral add up to 180°.
- *WHEN IN DOUBT…SKETCH IT OUT!*

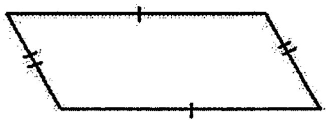
PROPERTIES OF PARALLELOGRAMS!!!

Parallelograms are always quadralaterals.

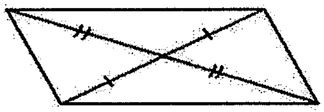


They must also have 2 pairs of parallel sides, thus giving it the name.

Opposite angles in a parallelogram are congruent.



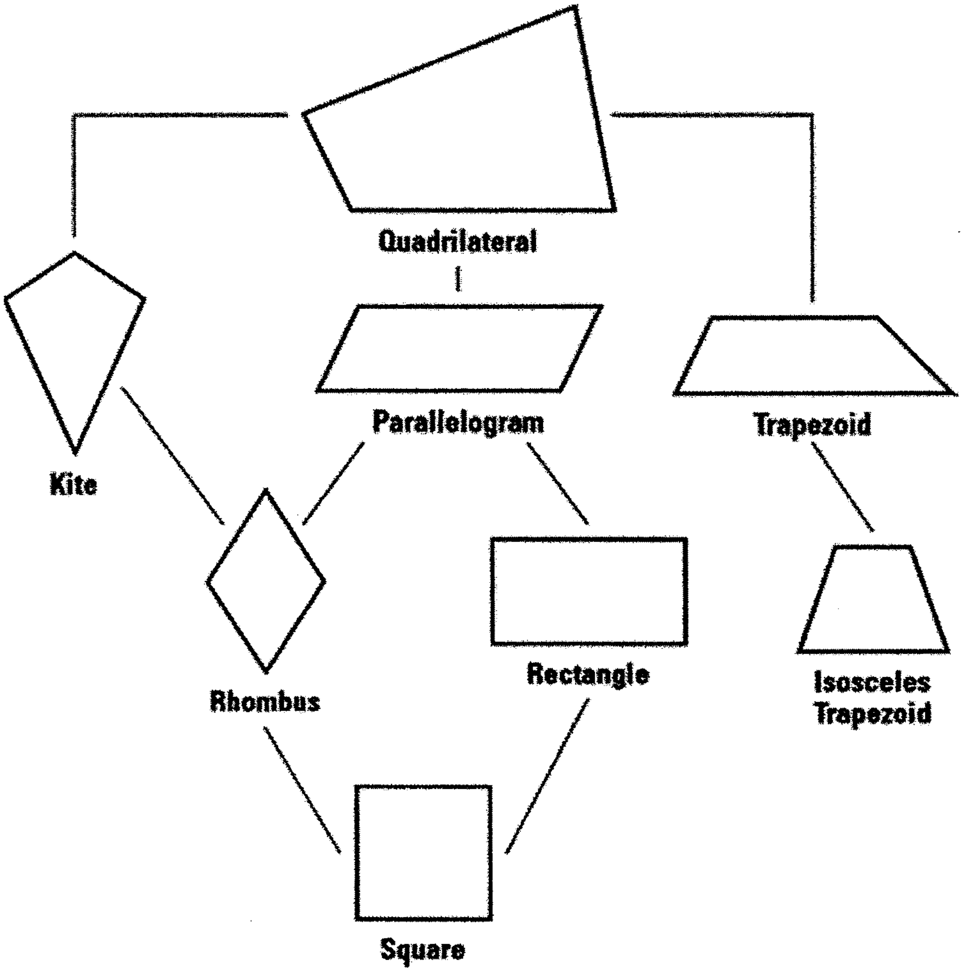
Opposite sides of a parallelogram are congruent.



The 2 diagonals in a parallelogram always bisect eachother.

Any 2 adjacent angles are also alternate interior angles.

PARALLELOGRAM FAMILY TREE!!!

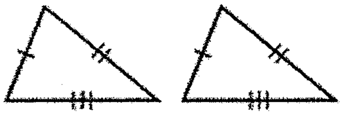


TRIANGLE CONGRUENCE

What are the five ways that we can prove that two triangles are congruent?!?!?!

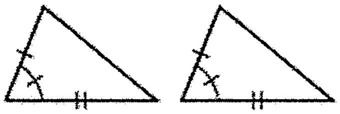
- SSS (Side-Side-Side)
- ASA (Angle-Side-Angle)
- SAS (Side-Angle-Side)
- AAS (Angle-Angle-Side)

Side-Side-Side (SSS)



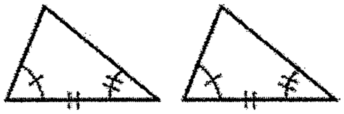
Three pairs of congruent sides

Side-Angle-Side (SAS)



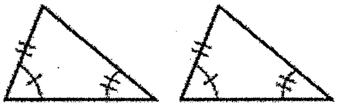
Two pairs of congruent sides and one pair of congruent angles (angles between the pairs of sides)

Angle-Side-Angle (ASA)



Two pairs of congruent angles and one pair of congruent sides (sides between the pairs of angles)

Side-Angle-Angle (SAA)



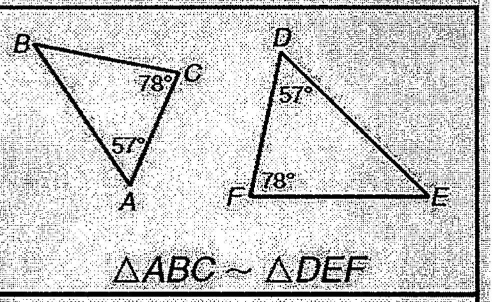
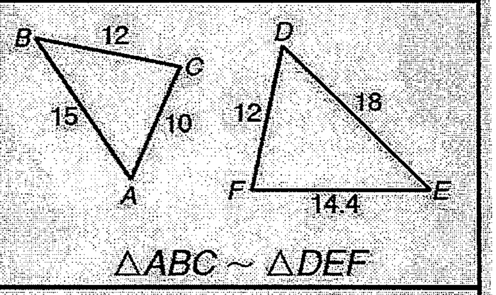
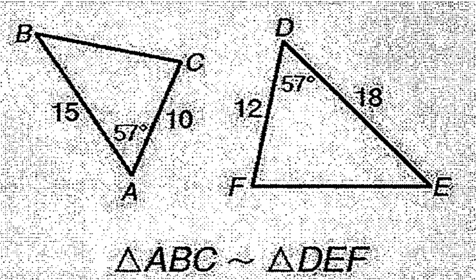
Two pairs of congruent angles and one pair of congruent sides (sides not between the pairs of angles)

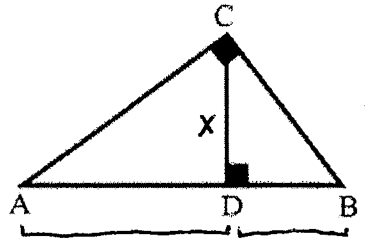
- HL (Hypotenuse-Leg) **Note: Can only be used with RIGHT triangles!**

TRIANGLE SIMILARITY

What are the three ways that we can prove that two triangles are similar?!?!?!

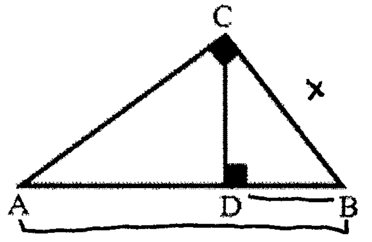
- AA (Angle-Angle)
- SAS (Side-Angle-Side)
- SSS (Side-Side-Side)

Angle-Angle (AA) Similarity	If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.	 $\triangle ABC \sim \triangle DEF$
Side-Side-Side (SSS) Similarity	If the three sides of one triangle are proportional to the three corresponding sides of another triangle, then the triangles are similar.	 $\triangle ABC \sim \triangle DEF$
Side-Angle-Side (SAS) Similarity	If two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.	 $\triangle ABC \sim \triangle DEF$



alt^2 = piece × piece

$x^2 = (AD)(DB)$

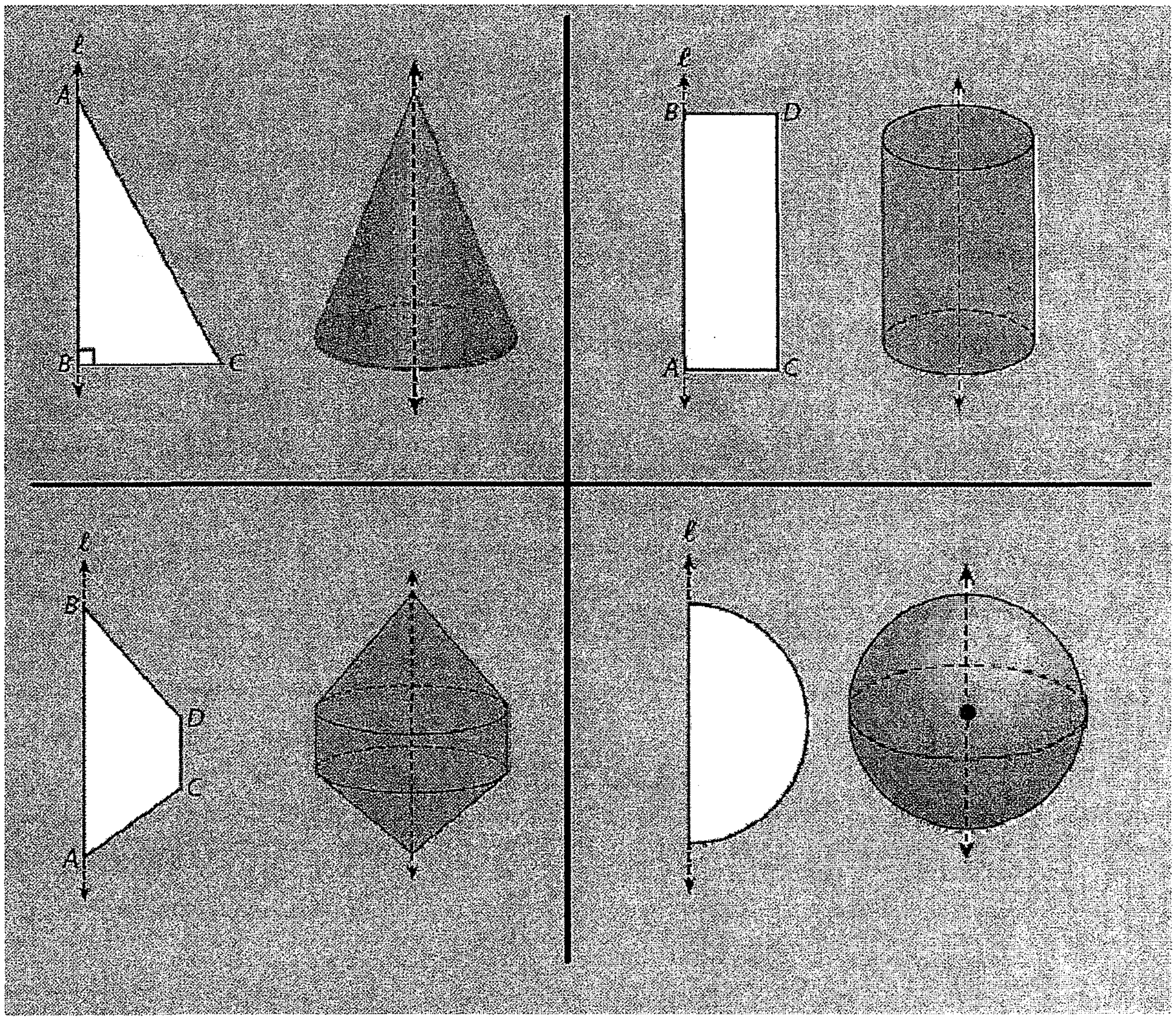


leg^2 = whole × piece

$x^2 = (AB)(DB)$

WHAT HAPPENS WHEN WE ROTATE TWO-DIMENSIONAL OBJECTS?!!?!

WE MAKE THREE-DIMENSIONAL OBJECTS!!!



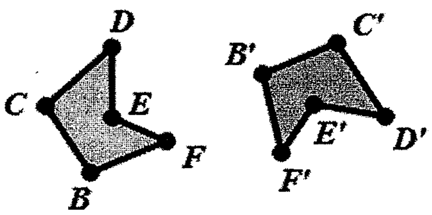
TRANSFORMATIONS

Rigid Transformations

<u>SIZE DOES NOT CHANGE!!!!!! DISTANCE IS PRESERVED!!!!</u>

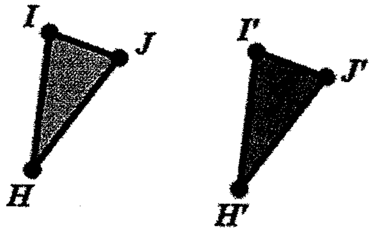
- Rotations
- Reflections
- Translations

Example #1



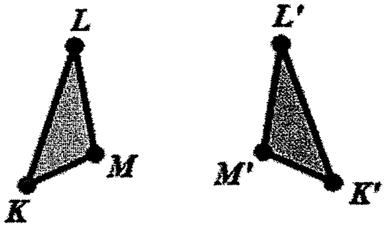
Rotate (Turn) – Example #1

Example #2



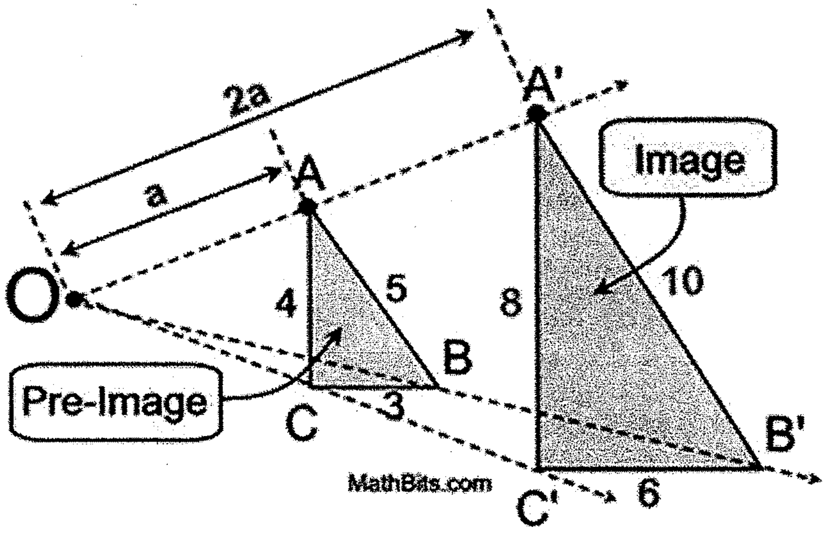
Translate (Slide) – Example #2

Example #3



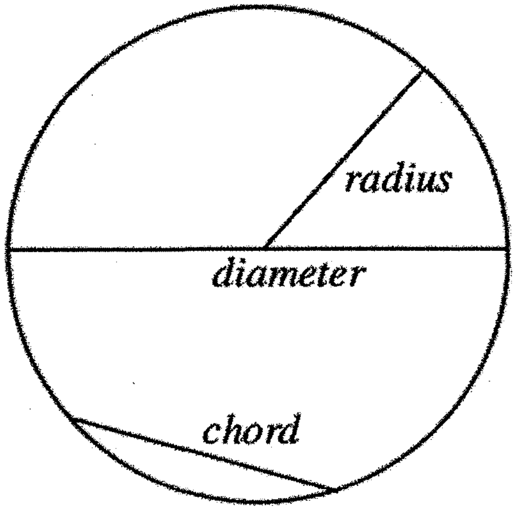
Reflection (Flip) - Example #3

DILATIONS!



Circle Geometry

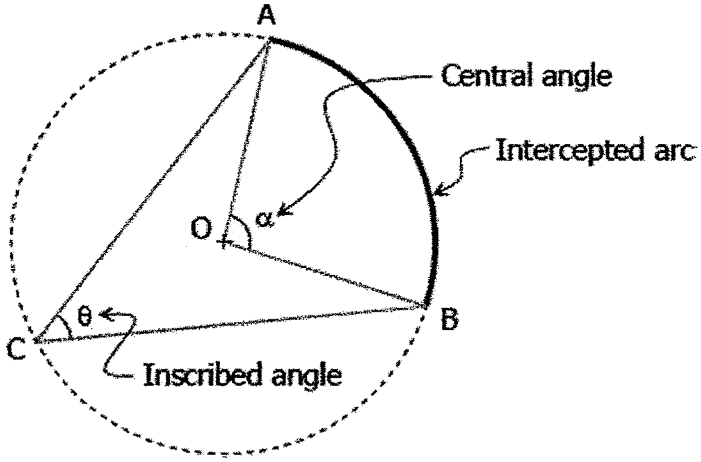
Circle Geometry Definitions



Radius: an interval joining centre to the circumference

Diameter: an interval passing through the centre, joining any two points on the circumference

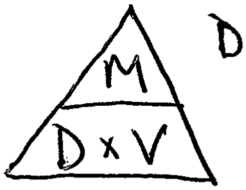
Chord: an interval joining two points on the circumference

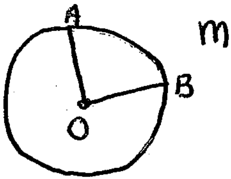
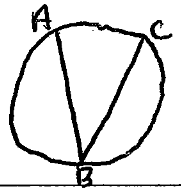
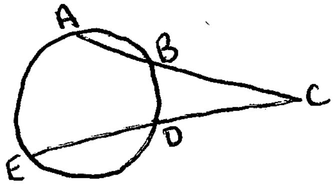
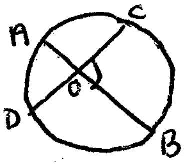
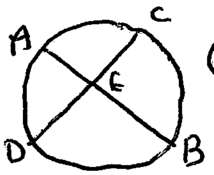
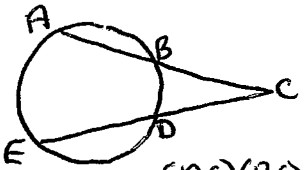
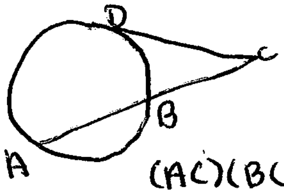


A central angle is an angle whose vertex is at the center of the circle. A central angle is ***equal*** *to the degree measure of the arc that it intercepts.*

An inscribed angle is an angle whose vertex is on the circle. An inscribed angle is ***half*** *of the degree measure of the arc that it intercepts.*

FORMULAS TO MEMORIZE!

NAME	FORMULA	KEY WORDS/MEMORIZATION TRICKS
Distance	$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Length, Congruent
Slope	$\frac{y_2-y_1}{x_2-x_1}$	Parallel lines have equal slopes Perpendicular lines have negative reciprocal slopes.
Midpoint	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$	Bisect
Density	 $D=\frac{m}{V}$	PER! DMV (the worst place in the world)
General Form of a Line	$y=mx+b$	M = slope B = Y-Intercept
Arc Length	$\frac{\theta}{360}\cdot 2\pi r$	Part of the circumference
Area of a Sector	$\frac{\theta}{360}\cdot \pi r^2$	Pizza slice (Part of the area)

General Form of a Circle	$(x-h)^2+(y-k)^2=r^2$	(h, k) = center (FLIP SIGNS!) R = Radius (Square Root)
Central Angle	central $\measuredangle$ = arc	Vertex on the center  $m\measuredangle AOB = m\widehat{AB}$
Inscribed Angle	inscribed $\measuredangle = \frac{1}{2}$ arc	Vertex on the circumference  $m\measuredangle ABC = \frac{1}{2} m\widehat{AC}$
Exterior Angle		Vertex outside the circle $\frac{\text{FARC} - \text{NARC}}{2}$
Interior Angle	 $\frac{\text{ARC}+\text{ARC}}{2}$	Vertex inside the circle (but not the center) $\measuredangle COB = \frac{\widehat{AD}+\widehat{CB}}{2}$
Chord-Chord	 $(AE)(EB) = (DE)(EC)$	Piece x Piece If you PP = PP
Secant-Secant	 $(AC)(BC)=(EC)(DC)$	Whole x Exterior When you WE = WE
Tangent-Secant	 $(AC)(BC)=(DC)^2$	Whole x Exterior = Tangent^2 Don't get WE = T^2

When you PP=PP with your WE=WE,
you don't want to get WE=T^2!

Area of a Rectangle	length x width	
Area of a Square	side x side	
Volume of a Rectangular Prism	l x w x h	BOX!
B	Upper case B stands for base area!	

TO PROVE A QUADRILATERAL IS A TRAPEZOID...

Property: A trapezoid has **one and only one pair** of opposite sides parallel.

Prove the Property: Use the slope formula 4 times to show that one and only one pair of opposite sides are parallel.

TO PROVE A QUADRILATERAL IS AN ISOSCELES TRAPEZOID...

Property: A trapezoid whose non-parallel sides are congruent.

Prove the Property: First, prove that the quadrilateral is a trapezoid by doing 4 slope formulas to show that one and only one pair of opposite sides are parallel. THEN, do the distance formula on the two **non-parallel** sides to show that they are congruent.

TO PROVE A QUADRILATERAL IS A PARALLELOGRAM...

Property: A parallelogram has **both** pairs of opposite sides parallel.

Prove the Property: Use the slope formula 4 times to show that both pairs of opposite sides are parallel.

TO PROVE A QUADRILATERAL IS A RECTANGLE...

Property: A parallelogram whose adjacent sides are perpendicular.

Prove Property: First, prove the quadrilateral is a parallelogram by doing 4 slope formulas to show that both pairs of opposite sides are parallel. THEN, look at the slopes that you already found and observe that the adjacent sides have negative reciprocal slopes, making them perpendicular!

TO PROVE A QUADRILATERAL IS A RHOMBUS...

Property: A parallelogram whose sides are all congruent.

Prove Property: Use the distance formula 4 times to show that all the sides are congruent.

TO PROVE A QUADRILATERAL IS A SQUARE...

Property: A parallelogram whose sides are all congruent and whose diagonals are congruent.

Prove Property: Use the distance formula 6 times to show that all sides are congruent AND the diagonals are congruent.

TO PROVE A QUADRILATERAL IS A KITE...

Property: A kite is a quadrilateral with both pairs of consecutive sides congruent.

Prove Property: Use the distance formula 4 times to show that both pairs of adjacent sides are congruent.

ANGLE, SEGMENT, & TRIANGLE RELATIONSHIPS & COORDINATE GEOMETRY

Polygons – Interior/Exterior Angles

Sum of Interior Angles: $180\ (n-2)$

Each Interior Angle of a Regular Polygon: $\frac{180\ (n-2)}{n}$

Sum of Exterior Angles: $360°$

Each Exterior Angle: $\frac{360}{n}$

Triangles

Classifying Triangles

Sides:

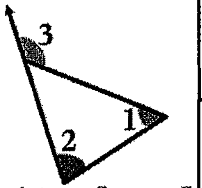
Scalene: No congruent sides
Isosceles: 2 congruent sides
Equilateral: 3 congruent sides

Angles:

Acute: All angles are < 90°
Right: One right angle that is 90°
Obtuse: One angle that is > 90°
Equiangular: 3 congruent angles (60°)

All triangles have 180°

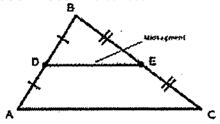
Exterior Angle Theorem: The exterior angle is equal to the sum of the two non-adjacent interior angles.



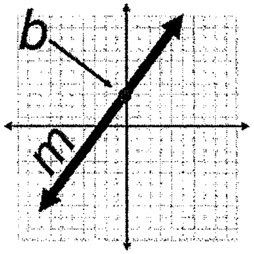
$m\angle 1 + m\angle 2 = m\angle 3$

Midsegment: a segment that joins two midpoints

- ➢ Always parallel to the third side
- ➢ $\frac{1}{2}$ the length of the third side
- ➢ Splits the triangle into two similar triangles



Coordinate Geometry

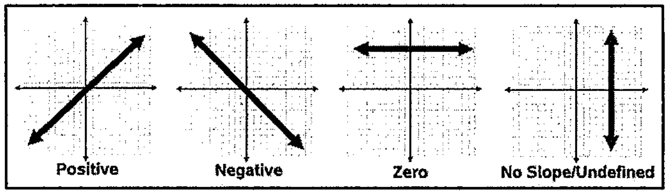


Slope-Intercept Form of a Line: $y = mx + b$ where m is the slope and b is the y-intercept.

Point-Slope Form of a Line: $y - y_1 = m(x - x_1)$ where m is the slope, and x_1 and y_1 are the values of a given point on the line.

Slope Formula: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Slopes:



- ➢ **Parallel** lines have the ***same*** slope
- ➢ **Perpendicular** lines have ***negative reciprocal*** slopes (flip the fraction & change the sign)
- ➢ **Collinear points** are points that lie of the ***same*** line.

Midpoint Formula: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Segment Ratios to Partition Line Segments:

$\frac{x - x_1}{x_2 - x} = \boldsymbol{Given\ Ratio}$ $\frac{y - y_1}{y_2 - y} = \boldsymbol{Given\ Ratio}$

Triangle Inequality Theorems

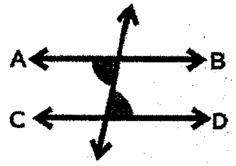
- ➢ The sum of 2 sides must be greater than the third side
- ➢ The difference of 2 sides must be less than the third side
- ➢ The longest side of the triangle is opposite the largest angle
- ➢ The shortest side of the triangle is opposite the smallest angle

Isosceles Triangle

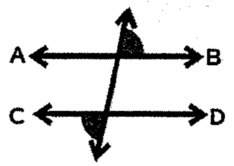
- ➢ 2 ≅ sides and 2 ≅ base angles
- ➢ The altitude drawn from the vertex is also the median and angle bisector
- ➢ If two sides of a triangle are ≅, then the angles opposite those ≅ sides are ≅

Parallel Lines

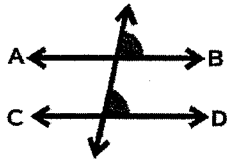
Alternate interior angles are **congruent**



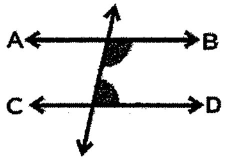
Alternate exterior angles are **congruent**



Corresponding angles are **congruent**

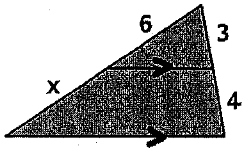


Same-side interior angles are **supplementary**



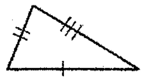
Side – Splitter Theorem

If a line is parallel to a side of a triangle and intersects the other two sides, then this line divides those two sides proportionally.

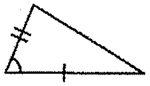


Triangle Congruence Theorems

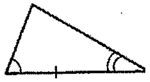
Side-Side-Side **(SSS)**



Side-Angle-Side **(SAS)**

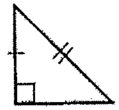


Angle-Side-Angle **(ASA)**



Angle-Angle-Side **(AAS)**

Hypotenuse-Leg **(HL)**

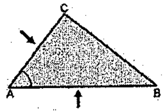


CPCTC – Corresponding Parts of Congruent Triangles are Congruent

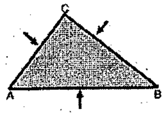
Similar Triangle Theorems

Angle-Angle **(aa)**

Side-Angle-Side **(SAS)**



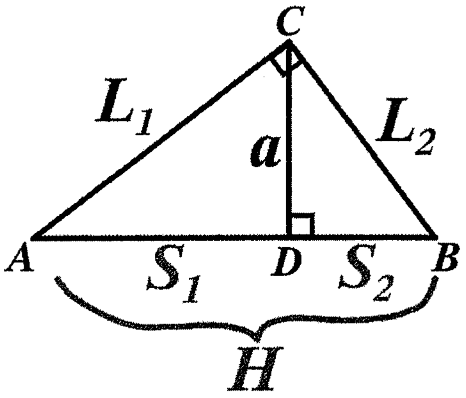
Side-Side-Side **(SSS)**



- Similar figures have congruent angles and proportional sides
- **CSSTP -** Corresponding Sides of Similar Triangles are in Proportion
- In a proportion, the product of the means equals the product of the extremes

The Mean Proportional

Altitude Theorem (SAAS / Heartbeat Method):
The altitude is the geometric mean between the 2 segments of the hypotenuse.



$$\frac{S_1}{a} = \frac{a}{S_2}$$

Leg Theorem (HYLLS / PSSW):
The leg is the geometric mean between the segment it touches and the whole hypotenuse.

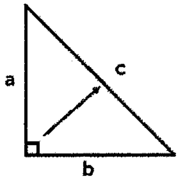
$$\frac{S_1}{L_1} = \frac{L_1}{H} \quad \text{and} \quad \frac{S_2}{L_2} = \frac{L_2}{H}$$

The Pythagorean Theorem

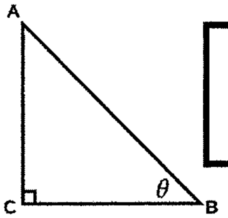
To find the missing side of any **right** triangle if two sides are given, use:

$$a^2 + b^2 = c^2$$

where *a* and *b* are the legs, and *c* is the hypotenuse



Trigonometry (SOHCAHTOA)



$\sin\theta = \frac{opposite}{hypotenuse}$	$\cos\theta = \frac{adjacent}{hypotenuse}$	$\tan\theta = \frac{opposite}{adjacent}$

- When solving for a side, use the sin, cos, and tan buttons
- When solving for an angle, use the $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ buttons
- *Recall from Algebra 1*: $\boldsymbol{Average\ Speed} = \frac{\Delta Distance}{\Delta Time}$ and $\boldsymbol{Speed} = \frac{Distance}{Time}$

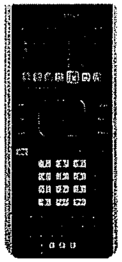
Cofunctions:

- Sine and Cosine are cofunctions, which are complementary

$$\mathbf{sin}\boldsymbol{\theta} = \mathbf{cos}(\mathbf{90°} - \boldsymbol{\theta})$$
$$\mathbf{cos}\boldsymbol{\theta} = \mathbf{sin}(\mathbf{90°} - \boldsymbol{\theta})$$

- If $\angle A$ and $\angle B$ are the acute angles of a right triangle, then $\mathbf{sin}\,A = \mathbf{cos}\,B$

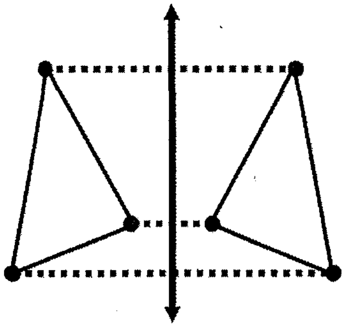
It's calculator time!



TRANSFORMATIONAL GEOMETRY

Rigid Motion: a type of transformation that preserves distance, congruency, angle measure, size, and shape.

Reflection – FLIP



$r_{x-axis}(x,y) = (x,-y)$

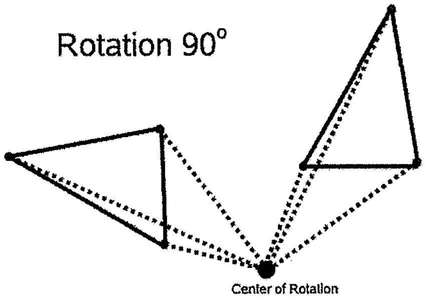
$r_{y-axis}(x,y) = (-x,y)$

$r_{y=x}(x,y) = (y,x)$

$r_{y=-x}(x,y) = (-y,-x)$

$r_{(o,o)}(x,y) = (-x,-y)$

Rotation – TURN

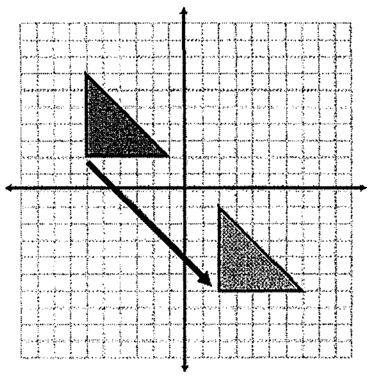


$R_{90°}(x,y) = (-y,x)$

$R_{180°}(x,y) = (-x,-y)$

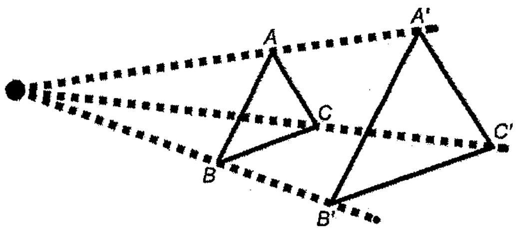
$R_{270°}(x,y) = (y,-x)$

Translation – SHIFT/MOVE



$T_{a,b}(x,y) = (x+a,y+b)$

Dilation – ENLARGEMENT/REDUCTION



$D_k(x,y) = (k \cdot x, k \cdot y)$

- **Dilations create similar figures, where the corresponding sides are in proportion and the corresponding angles are congruent.**
- **Dilations are *not always* rigid motions, since they do *not always* preserve distance or congruency.**

Composition of Transformations

When you see "∘", work from right to left.

$$R_{90°} \circ T_{3,-4}$$

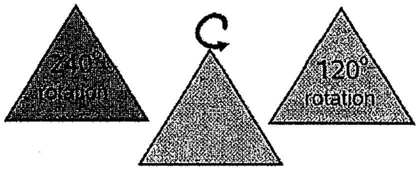
The example shows a translation to the right by three units and down by four units, followed by a rotation of 90 degrees.

Types of Composition Transformations

- A composition of 2 reflections over 2 parallel lines is equivalent to a **translation.**
- A composition of 2 reflections over 2 intersecting lines is equivalent to a **rotation.**

Rotational Symmetry Theorem

A regular polygon with n sides always has rotational symmetry, with rotations in increments equal to its central angle of $\frac{360°}{n}$. Rotational symmetry is commonly referred as "mapping the figure onto itself".



CIRCLES

Circle Definition: A 2-dimensional shape made by drawing a curve that is always the same distance from the center.

Circle Equations

General/Standard Equation of a Circle:

$$\boldsymbol{x^2 + y^2 + Cx + Dy + E = 0}$$

where C, D, and E are constants.

Center – Radius Equation of a Circle:

$$\boldsymbol{(x - h)^2 + (y - k)^2 = r^2}$$

where *(h, k)* is the center and *r* is the radius.

Completing the Square

The method of "completing the square" is used when factoring by the basic "Trinomial Method", or "AM" method cannot be applied to the problem. The completing the square method is commonly used in geometry ***to express a general circle equation in center-radius form.***

Example: Express the general equation $x^2 + 4x + y^2 - 6y - 12 = 0$ in center-radius form.

$$x^2 + 4x + y^2 - 6y - 12 = 0$$

$$x^2 + 4x + y^2 - 6y = 12$$

$$x^2 + 4x + \_ + y^2 - 6y + \_ = 12 + \_ + \_$$

$$x^2 + 4x + \mathbf{4} + y^2 - 6y + \mathbf{9} = 12 + \mathbf{4} + \mathbf{9}$$

$$(x + 2)(x + 2) + (y - 3)(y - 3) = 25$$

$$\boxed{(x + 2)^2 + (y - 3)^2 = 25}$$

Formula: $\left(\frac{b}{2}\right)^2$

Steps:

1) Determine if the squared terms have a coefficient of 1
2) If there is a constant/number on the left side of the equal sign, move that constant to the right side
3) Insert "boxes" or "blank spaces" after the linear terms to acquire a perfect-square trinomial
4) Take half of the linear term(s) and square the number. Insert this number on both the left and right sides
5) Factor using the "trinomial method"
6) Write your equation

Review of Factoring

The order of Factoring:

Greatest Common Factor **(GCF)**

↓

Difference of Two Perfect Squares **(DOTS)**

↓

Trinomial/"AM Method" **(TRI)**

GCF:

$$ab + ac = a(b + c)$$

DOTS:

$$x^2 - y^2 = (x + y)(x - y)$$

TRI:

$x^2 - x + 6$ » $(x + 2)(x - 3)$

Graphing Circles

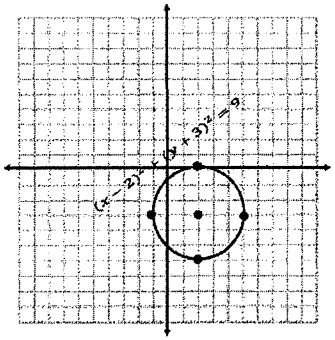
Steps:

1) Determine the center and the radius
2) Plot the center on the graph
3) Around the center, create four loci points that are equidistant from the center of the circle
4) Using a compass or steady freehand, connect all four points
5) Label when finished

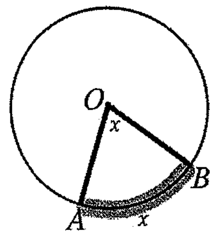
Example: Graph $(x - 2)^2 + (y + 3)^2 = 9$

The center is the point (2,-3)

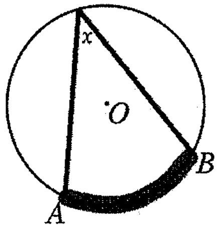
The radius is 3



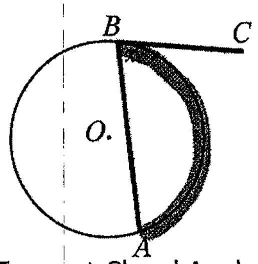
Angle Relationships in a Circle



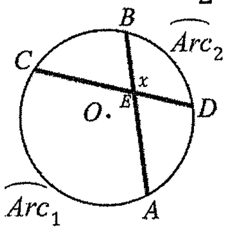
Central Angle:
$\measuredangle x = \widehat{AB}$



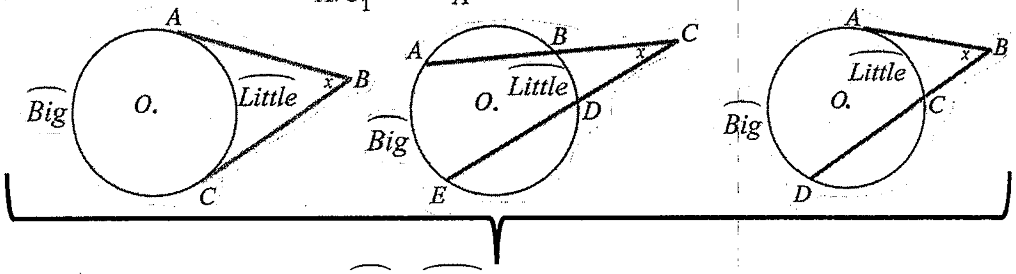
Inscribed Angle:
$\measuredangle x = \frac{1}{2}\widehat{AB}$



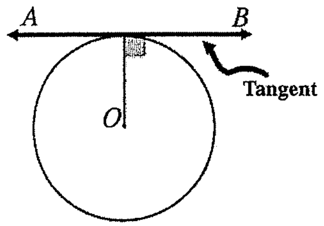
Tangent-Chord Angle:
$\measuredangle x = \frac{1}{2}\widehat{AB}$



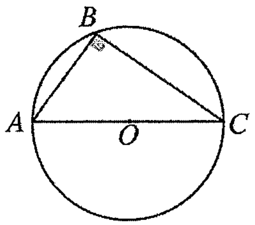
Two Chord Angles:
$\measuredangle x = \frac{\widehat{Arc_1} + \widehat{Arc_2}}{2}$



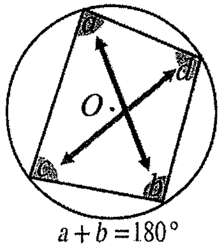
$\frac{\widehat{Big} - \widehat{Little}}{2} = \measuredangle x$



A tangent is perpendicular to its radius, forming a 90° angle



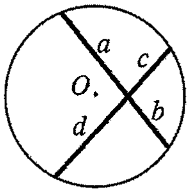
An angle that is inscribed in a semicircle equals 90°



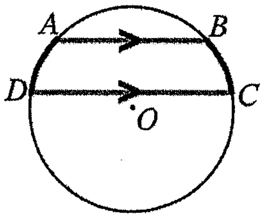
$a + b = 180°$
$c + d = 180°$

If a quadrilateral is inscribed in a circle, then its opposite angles = 180°

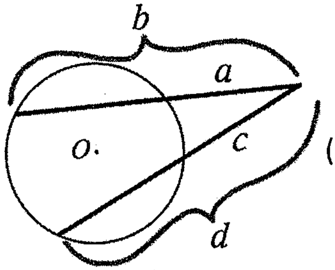
Segment Relationships in a Circle



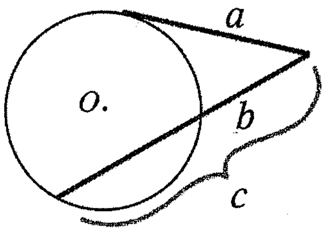
(Part)(Part)=(Part)(Part)
$(\boldsymbol{a})(\boldsymbol{b}) = (\boldsymbol{c})(\boldsymbol{d})$



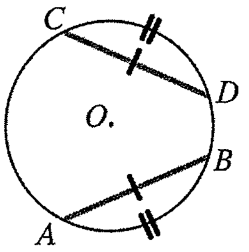
If $\overline{AB} \parallel \overline{DC}$, then $\widehat{AD} \cong \widehat{BC}$
Parallel chords intercept congruent arcs



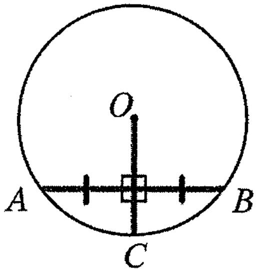
(W)(E) = (W)(E)
(Whole)(External)=(Whole)(External)
$(\boldsymbol{b})(\boldsymbol{a}) = (\boldsymbol{d})(\boldsymbol{c})$



(W)(E) = $(T)^2$
(Whole)(External)=$(\text{Tangent})^2$
$(\boldsymbol{c})(\boldsymbol{b}) = (\boldsymbol{a})^2$



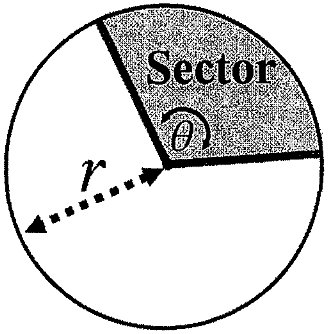
If $\overline{AB} \cong \overline{CD}$, then $\widehat{AB} \cong \widehat{CD}$



If a diameter/radius is perpendicular to a chord, then the diameter/radius bisects the chord and its arc.

Circles (Con't)

Area of a Sector



$$A = \frac{1}{2}r^2\theta$$

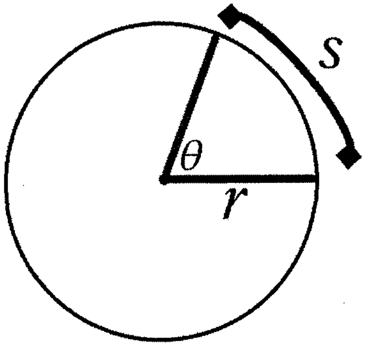
where A is the area of the sector, r is the radius, and θ is an angle in radians.

-or-

$$A = \frac{n}{360}\pi r^2$$

where A is the area of the sector, n is the amount of degrees in the central angle, and r is the radius

Sector Length

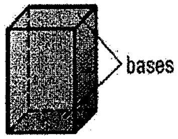


$$s = r \cdot \theta$$

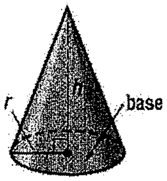
where s is the sector length, r is the radius, and θ is an angle in radians.

3-D FIGURES

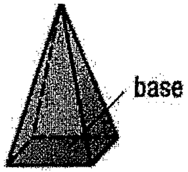
Prism



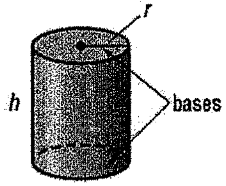
Cone



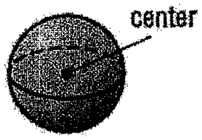
Pyramid



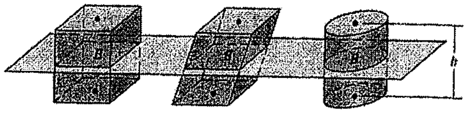
Cylinder



Sphere



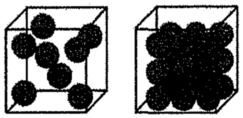
Cavalieri's Principle: If two solids have the same height and the same cross-sectional area at every level, then the solids have the same volume.



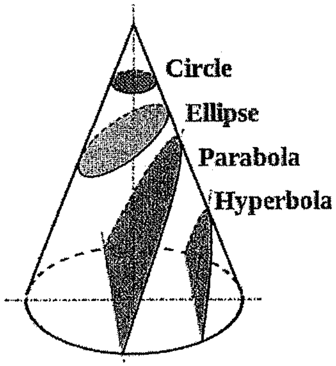
Density Formulas:

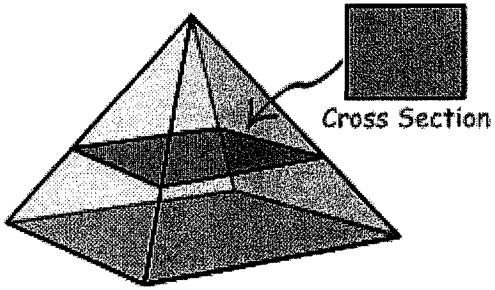
$$Mass = (Density) \cdot (Volume)$$

$$Density = \frac{(Mass)}{(Volume)}$$



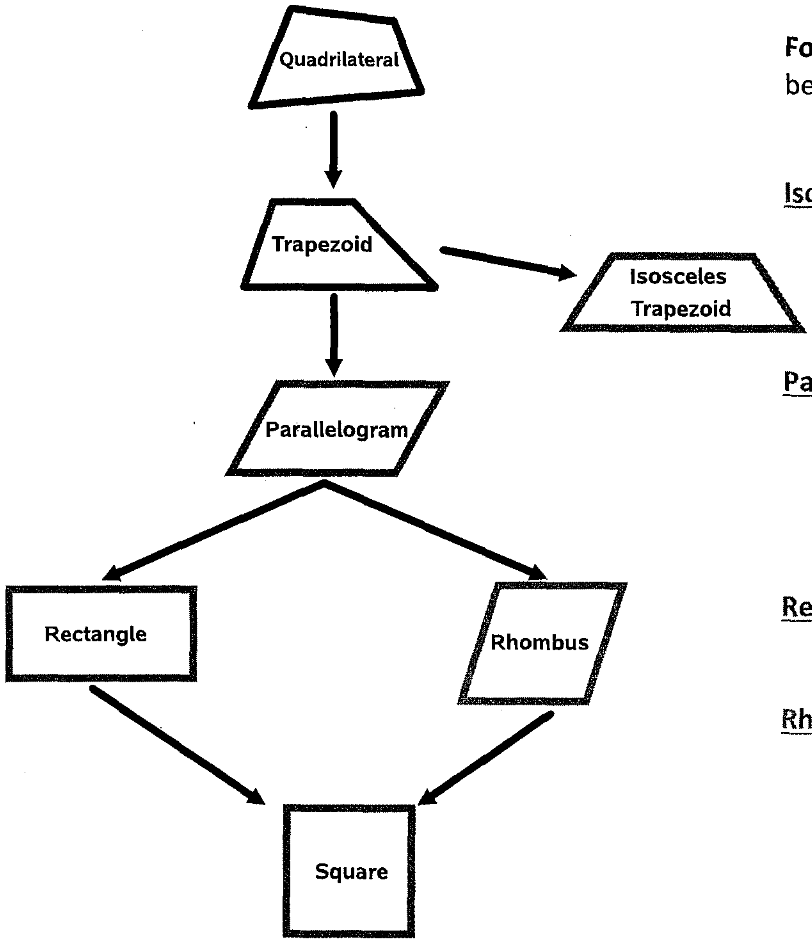
Cross Sections: a surface or shape that is or would be exposed by making a straight cut through something at one or multiple points.





QUADRILATERALS

The Quadrilateral Family Tree



The Quadrilateral Properties

Quadrilateral

- ✓ A quadrilateral is a four-sided polygon

Trapezoid

- ✓ at least one pair of parallel sides

Formula: The length of the **median** of a trapezoid can be calculated using the following formula:

$$Median = \frac{1}{2}(Base_1 + Base_2)$$

Isosceles Trapezoid

- ✓ each pair of base angles are congruent
- ✓ diagonals are congruent
- ✓ one pair of congruent sides (which are the called the *legs*. These are the non-parallel sides)

Parallelogram

- ✓ opposite sides are parallel
- ✓ opposite sides are congruent
- ✓ opposite angles are congruent
- ✓ consecutive angles are supplementary
- ✓ diagonals bisect each other

Rectangle

- ✓ all angles at its vertices are right angles
- ✓ diagonals are congruent

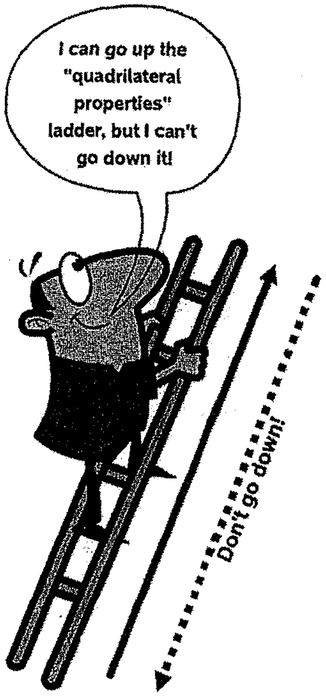
Rhombus

- ✓ all sides are congruent
- ✓ diagonals are perpendicular
- ✓ diagonals bisect opposite angles
- ✓ diagonals form four congruent right triangles
- ✓ diagonals form two pairs of two congruent isosceles triangles

Square

- ✓ diagonals form four congruent isosceles right triangles

Each figure *inherits the properties* of its parent



COORDINATE GEOMETRY PROOFS WITH POLYGONS

How to prove Quadrilaterals

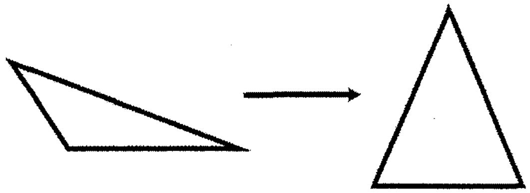
- To prove that a *quadrilateral* is a *parallelogram*, it is sufficient to show any one of these properties:
  - ✓ Both pairs of opposite sides are parallel
  - ✓ Both pairs of opposite sides are congruent
  - ✓ Both pairs of opposite angles are congruent
  - ✓ One pair of opposite sides are both parallel and congruent
  - ✓ Diagonals bisect each other

- To prove that a *parallelogram* is a *rectangle*, it is sufficient to show any one of these:
  - ✓ Any one of its angles is a right angle
  - ✓ One pair of consecutive angles are congruent
  - ✓ Diagonals are congruent

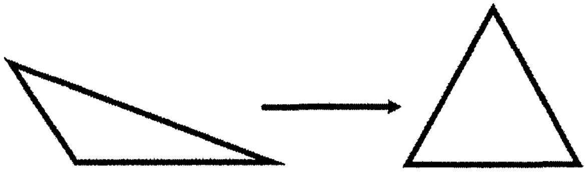
- To prove that a *parallelogram* is a *rhombus*, it is sufficient to show any one of these:
  - ✓ One pair of consecutive sides are congruent
  - ✓ Diagonals are perpendicular
  - ✓ Either diagonal is an angle bisector

How to prove Triangles

- To prove that a given triangle is an *isosceles* triangle, it is sufficient to show that two sides are congruent.



- To prove that a given triangle is an *equilateral* triangle, it is sufficient to show that all three sides are congruent.

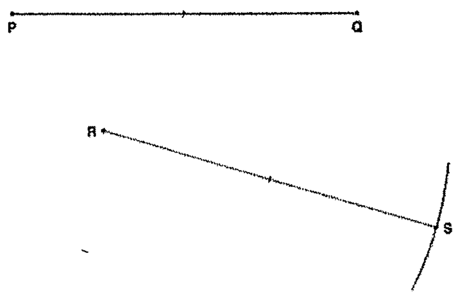


Remember – if there is a coordinate geometry proof on the regents, devise a plan, write it down, and use the coordinate geometry formulas shown in the "Coordinate Geometry" section of this packet to prove some properties!

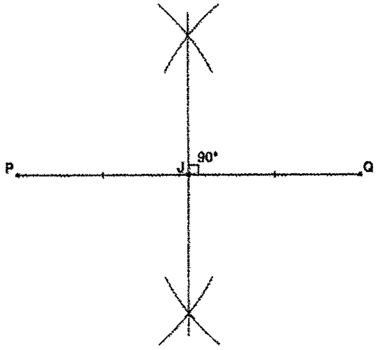
CONSTRUCTIONS

There will be either one or two constructions on the Geometry regents. It is important to understand basic constructions.

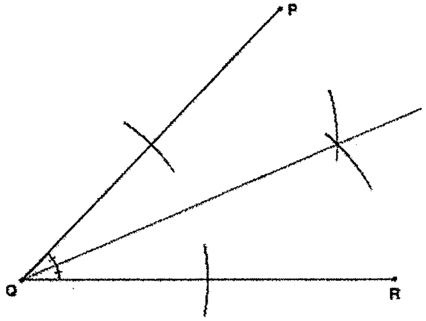
Copy a Line Segment



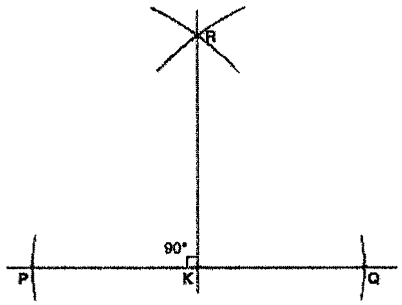
Perpendicular Bisector



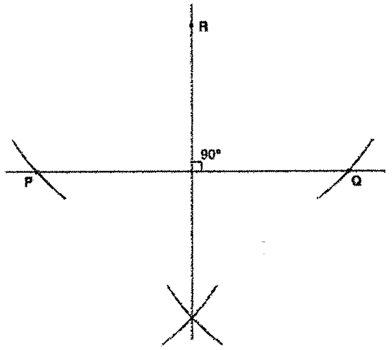
Angle Bisector



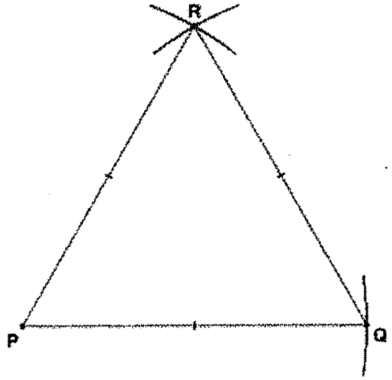
Perpendicular Line passing through a Point on the Given Line



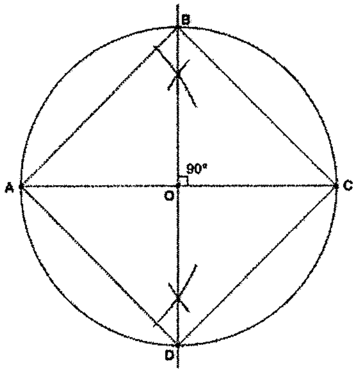
Perpendicular Line passing through a Point NOT on the Given Line



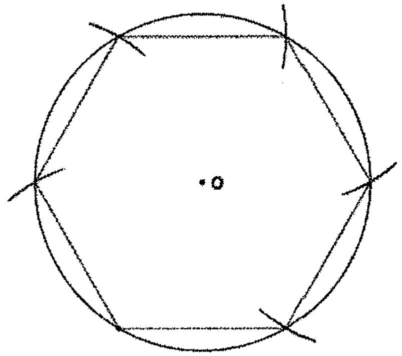
Equilateral Triangle



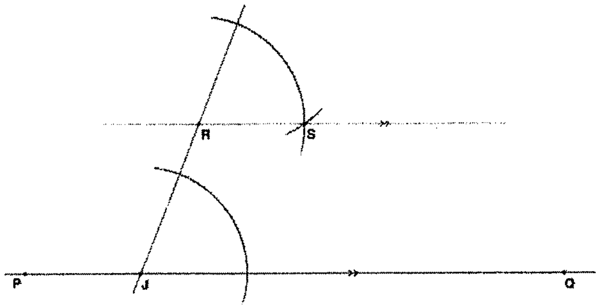
Square Inscribed in a Circle



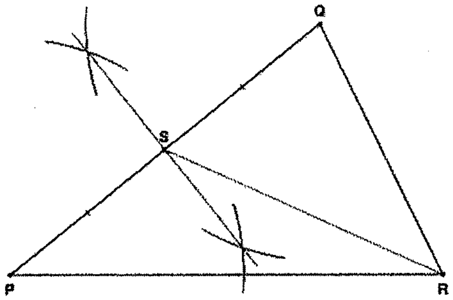
Hexagon Inscribed in a Circle



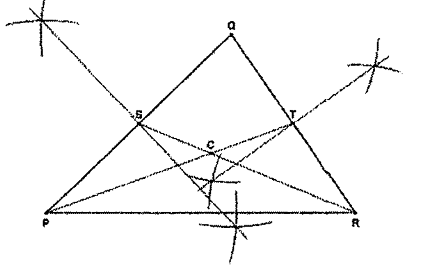
Parallel Lines



Median – A median is drawn to its midpoint



Centroid – The intersection of 3 medians



Geometry Comr Core Study Guide

Polygon Interior/Exterior Angles

Sum of interior angles = 180(n – 2)

Each interior angle (regular) = $\frac{180(n-2)}{n}$

Sum of exterior angles = 360°

Each exterior angle (regular) = $\frac{360}{n}$

Triangles

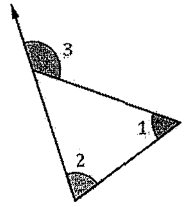
Classifying:

Sides: Scalene = no congruent sides
Isosceles = 2 congruent sides
Equilateral = 3 equal sides

Angles: Acute = all angles < 90°
Right = one right angle of 90°
Obtuse = one angle > 90°
Equiangular = 3 congruent angles (60°)

Sum of the angles of any triangle = 180°

Exterior angle = sum of 2 non-adjacent interior angles



m∠1 + m∠2 = m∠3

Midsegment – segment joining the midpoints

- **Always parallel to the third side**
- **½ the length of the third side**
- **Splits the triangle into similar triangles**

Pythagorean Theorem

To find a missing side of a right triangle:

$$a^2 + b^2 = c^2$$

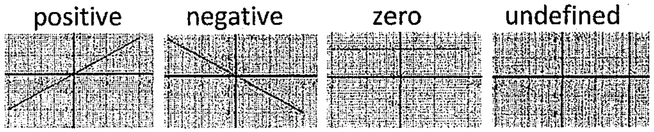
Where c is the hypotenuse

Supplementary Angles – sum to 180°
Complementary Angles – sum to 90°

Coordinate Geometry

Linear $y = mx + b$ where m=slope & b=y-intercept

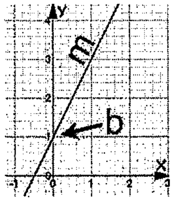
slope = $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$



Collinear – on the same line

Parallel lines have the same slope

Perpendicular lines have negative reciprocal slopes
(flip and change the sign)

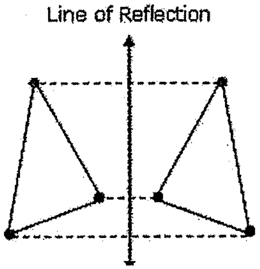


Midpoint = $\left(\frac{x1+x2}{2}, \frac{y1+y2}{2}\right)$

Distance = $\sqrt{(x1 - x2)^2 + (y1 - y2)^2}$

Rigid Motion – transformations that preserve size and shape (Isometric)

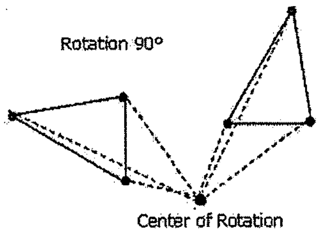
Reflection - FLIP



$r_{x\text{-axis}}(x, y) = (x, -y)$
$r_{y\text{-axis}}(x, y) = (-x, y)$
$r_{y=x}(x, y) = (y, x)$
$r_{y=-x}(x, y) = (-y, -x)$
$r_{origin}(x, y) = (-x, -y)$

Opposite isometry- orientation NOT preserved

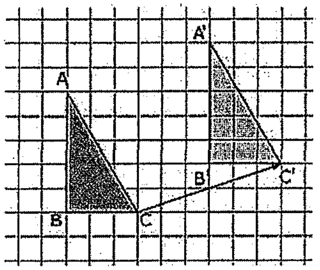
Rotation - TURN



$R_{90°}(x, y) = (-y, x)$
$R_{180°}(x, y) = (-x, -y)$
$R_{270°}(x, y) = (y, -x)$

Direct Isometry- preserves orientation

Translation – SLIDE



$T_{a,b}(x, y) = (x+a, y+b)$

Composition of Transformations

$R_{90°} \circ T_{3,-4}$

←

Translation followed by a rotation

Factoring

Look for GCF first!!

GCF: $ab + ac = a(b + c)$

Difference Of Perfect Squares:
$x^2 - y^2 = (x + y)(x - y)$

Trinomial: $2x^2 + 7x - 15 =$
$(2x - 3)(x + 5)$
-3x
+10x
+7x

Similar Triangles Theorems

Similar figures have congruent angles and proportional sides

AA; SSS; SAS Similarity Thm

-Corresponding sides of similar triangles are in proportion

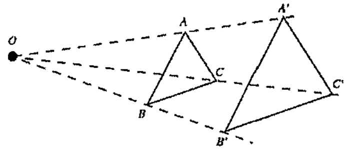
-In a proportion, the product of the means = the product of the extremes

Congruence Triangle Theorems

SAS; AAS; ASA; SSS; HL ≅ Thm

CPCTC (CPCFC)

Dilation – NOT rigid motion, NOT isometry



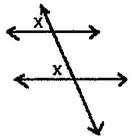
$D_k(x, y) = (kx, ky)$

Dilations create similar figures.

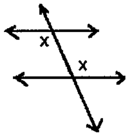
A composition of 2 reflections over 2 parallel lines is equivalent to a translation.
A composition of 2 reflections over 2 intersecting lines is equivalent to a rotation.

Parallel Lines cut by a transversal

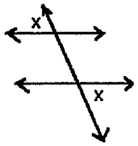
Corresponding Angles are ≅



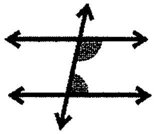
Alternate Interior angles are ≅



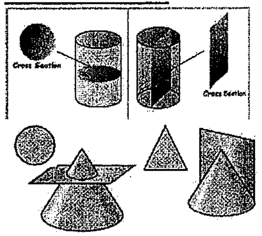
Alternate exterior angles are ≅



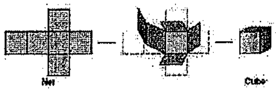
Interior angle on the same side are supplementary



Cross-sections



Nets



Coordinate geometry proofs

Slope formula:

- Parallel lines
- Perpendicular lines
- Right angle

Midpoint Formula:

- Segments bisect each other

Distance Formula:

- Length

Isosceles Triangle

2 congruent sides & 2 congruent base angles

The altitude from the vertex is also the median and angle bisector.

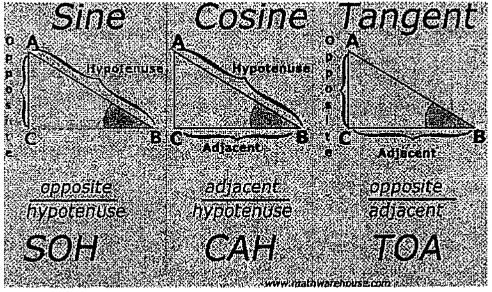
Triangle Inequality

- The sum of the lengths of any 2 sides must be greater than the 3rd
- The longest side is opposite the largest angle
- The measure of the exterior angle is greater than either non-adjacent interior angle

Side Splitter Theorem

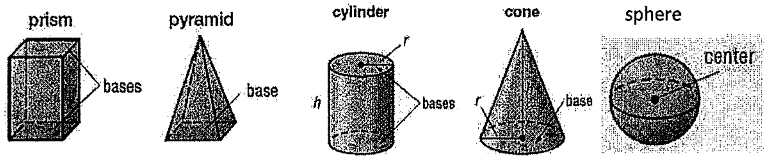
A line drawn parallel to any of the sides of a triangles divides the other 2 sides proportionally

Trigonometry

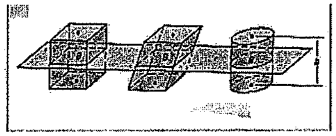


When solving for an angle used the inverse (2nd Key)

3D Solids



Cavalieri's Principle – If 2 solids have the same height and the same cross-sectional area at every level, they have the same volume

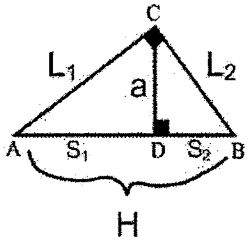


Mass = **(Volume) (Density)**

Geometric Mean Theorems

Altitude Theorem – The altitude is the geometric mean between the 2 segments of the hypotenuse. $\frac{S1}{a} = \frac{a}{S2}$

Leg Theorem – The leg is the geometric mean between the segment it touches and the whole hypotenuse $\frac{S1}{L1} = \frac{L1}{H}$ and $\frac{S2}{L2} = \frac{L2}{H}$

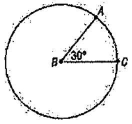


Circles

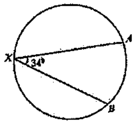
General Form Equation of a Circle
$x^2 + y^2 + Ax + By + C = 0$

Standard center radius form
$(x - h)^2 + (y - k)^2 = r^2$

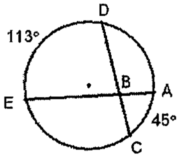
Angles



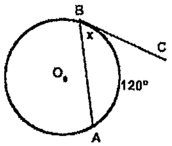
Central angle = arc



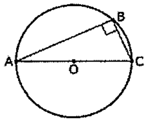
Inscribed angle = ½ arc



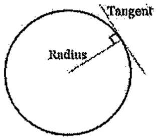
2 chords with vertical angles = $\frac{arc1+arc2}{2}$



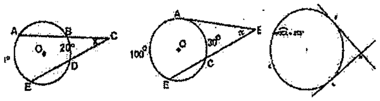
Tangent-chord angle = ½ arc



Angle inscribed in semicircle =

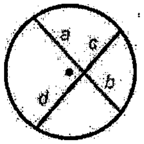


Tangent-radius angle = 90°

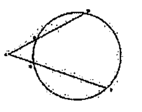


Angle outside circle formed by tangents and secants = $\frac{arc1-arc2}{2}$

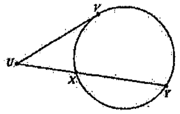
Segments



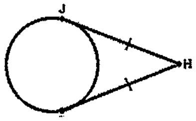
ab=cd



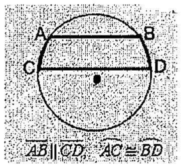
(Outside)(Whole) = (outside)(Whole)

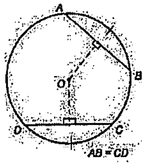


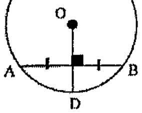
(Outside)(Whole) = (tangent)^2

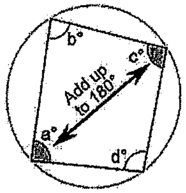


Tangents ≅



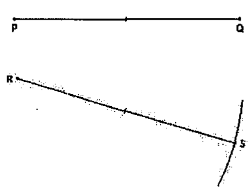




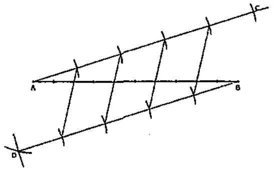


Basic Constructions

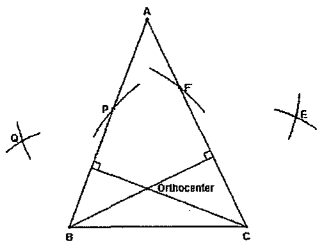
Copy a line segment



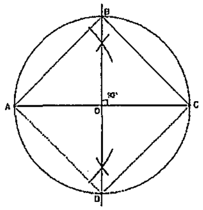
Dividing a segment into equal parts



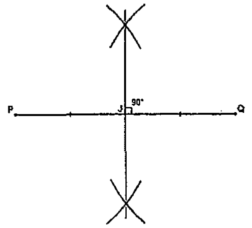
Orthocenter - altitudes



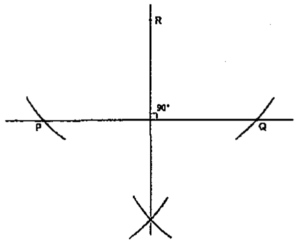
Inscribed square in a circle



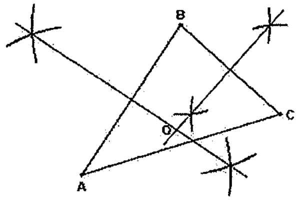
Perpendicular Bisector



Perpendicular line thru an external point

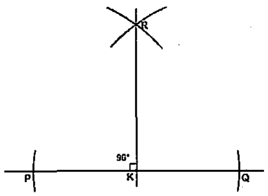


Circumcenter – perpendicular bisectors

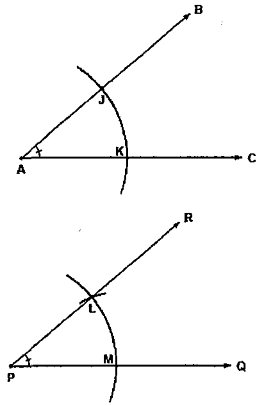


- Equidistant to each vertex of the triangle
- Used to circumscribe a circle

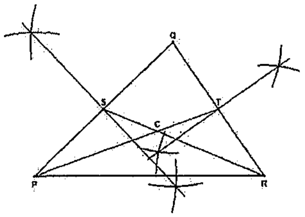
Perpendicular Line thru a point on a line



Copy an angle



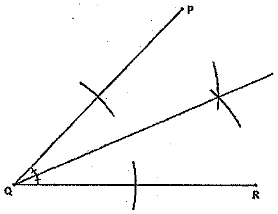
Centroid - medians



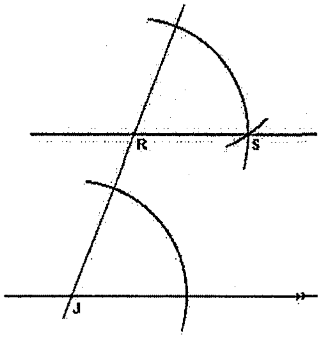
- 2:1 ratio

$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

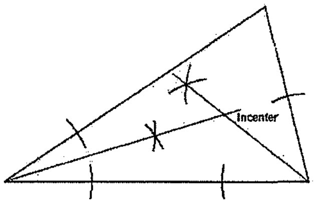
Bisect an angle



Parallel line

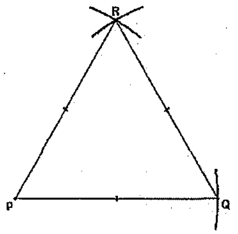


Incenter – angle bisectors

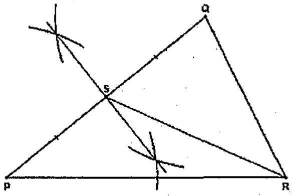


- equidistant to each side of the triangle
- Used to inscribe a circle

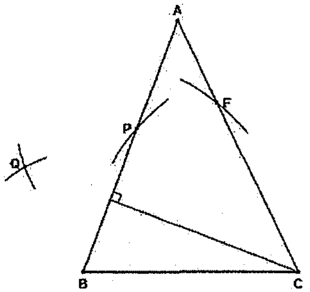
Equilateral Triangle

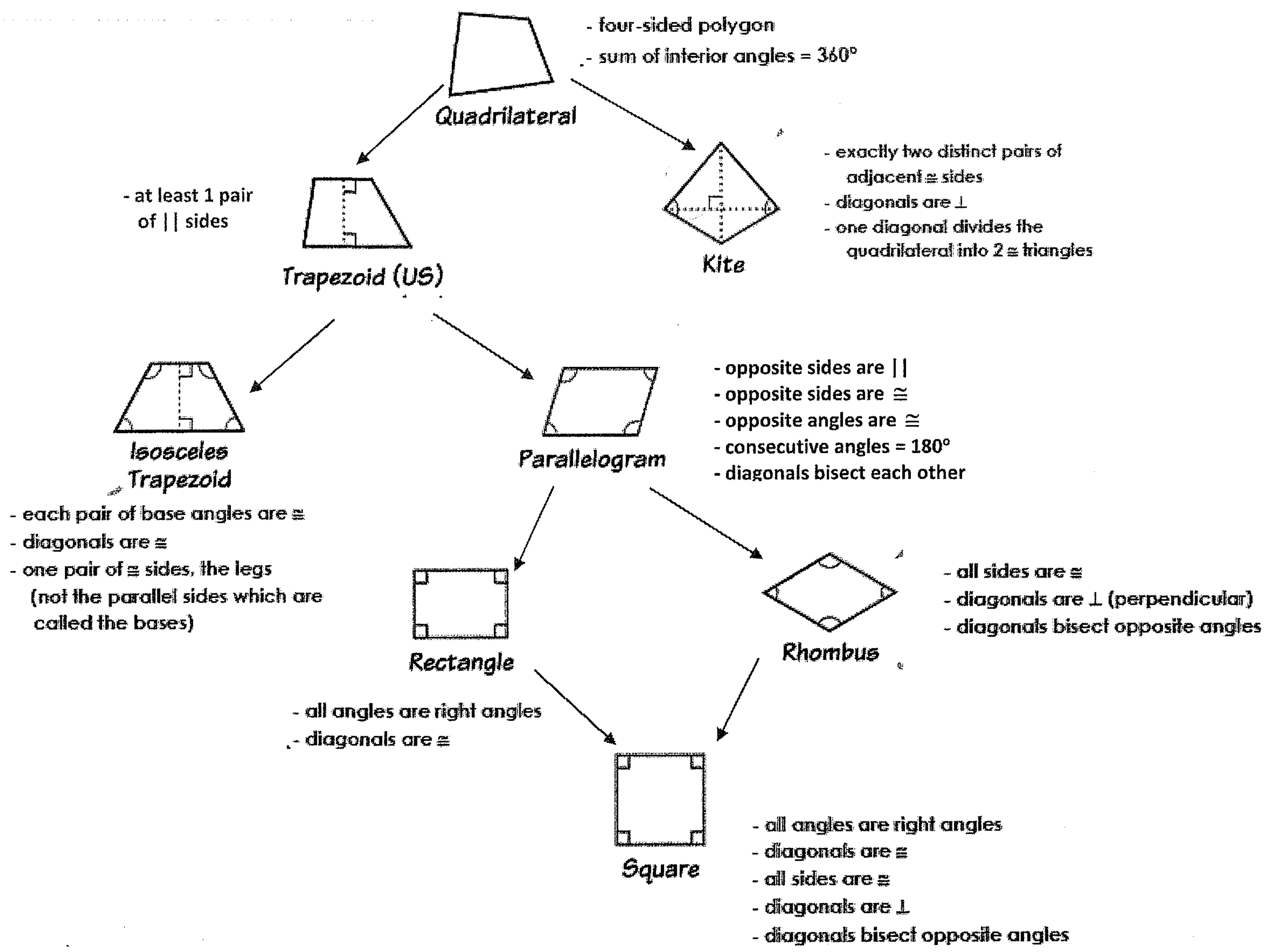


Median – vertex to midpoint



Altitude – vertex perpendicular to opposite side





Quadrilateral
- four-sided polygon
- sum of interior angles = 360°

Trapezoid (US)
- at least 1 pair of || sides

Kite
- exactly two distinct pairs of adjacent ≅ sides
- diagonals are ⊥
- one diagonal divides the quadrilateral into 2 ≅ triangles

Isosceles Trapezoid
- each pair of base angles are ≅
- diagonals are ≅
- one pair of ≅ sides, the legs (not the parallel sides which are called the bases)

Parallelogram
- opposite sides are ||
- opposite sides are ≅
- opposite angles are ≅
- consecutive angles = 180°
- diagonals bisect each other

Rectangle
- all angles are right angles
- diagonals are ≅

Rhombus
- all sides are ≅
- diagonals are ⊥ (perpendicular)
- diagonals bisect opposite angles

Square
- all angles are right angles
- diagonals are ≅
- all sides are ≅
- diagonals are ⊥
- diagonals bisect opposite angles

Coordinate Proofs:

Trapezoid:
- 1 pair of || sides

Isosceles Trapezoid:
Trapezoid +
- 2 ≅ sides OR
- 2 ≅ diagonals OR

Parallelogram: (pick 1)
- 2 pairs of || sides
- 2 pairs of ≅ sides
- Diagonals bisect each other
- 1 pair of || & ≅ sides

Rectangle:
- 4 right angles OR

Parallelogram +
- 1 right angle OR
- ≅ diagonals

Rhombus:
- 4 ≅ sides OR

Parallelogram +
- 2 consecutive ≅ sides OR
- ⊥ diagonals

Square:
- 4 right ∠'s and 2 consecutive ≅ sides OR
- 4 ≅ sides and 1 right ∠

Kite:
- 2 distinct pairs of ≅ sides