

GEOMETRY (COMMON CORE)

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY (Common Core)

Wednesday, August 12, 2015 — 8:30 to 11:30 a.m., only

Student Name: Answer Key ♡

School Name: MAP! ♡

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

GEOMETRY (COMMON CORE)

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. [48]

rhombus

1 A parallelogram must be a rectangle when its

- (1) diagonals are perpendicular
- (2) diagonals are congruent
- (3) opposite sides are parallel
- (4) opposite sides are congruent

what is special about a rectangle?

Use this space for computations.

properties of all parallelograms

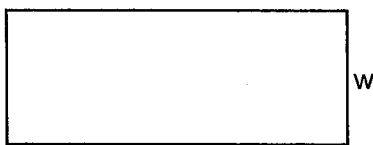
2 If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles not be congruent?

- (1) reflection over the x-axis
- (2) translation to the left 5 and down 4
- (3) dilation centered at the origin with scale factor 2
- (4) rotation of 270° counterclockwise about the origin

Reflection
Translation
Rotation } Rigid Transformations

similar Δ s!

3 If the rectangle below is continuously rotated about side w , which solid figure is formed?



- (1) pyramid
- (2) rectangular prism
- (3) cone
- (4) cylinder

When we rotate this rectangle around side w , what 3-D figure does it make?

4 Which expression is always equivalent to $\sin x$ when $0^\circ < x < 90^\circ$?

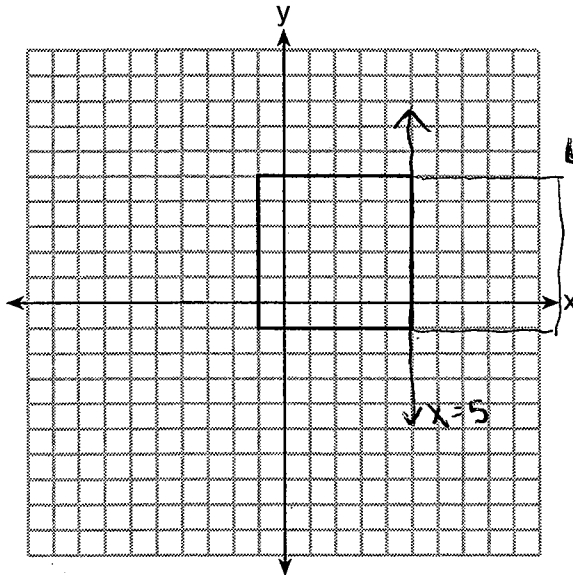
- (1) $\cos(90^\circ - x)$
- (2) $\cos(45^\circ - x)$
- (3) $\cos(2x)$
- (4) $\cos x$

complementary!

Add up to 90°

Use this space for computations.

5 In the diagram below, a square is graphed in the coordinate plane.

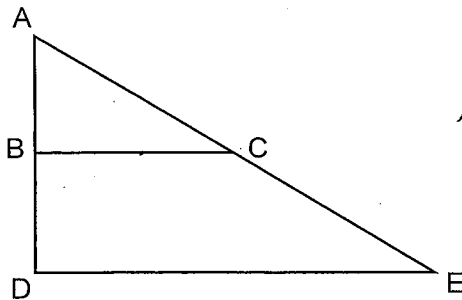


A reflection over which line does not carry the square onto itself?

- (1) $x = 5$
- (2) $y = 2$
- (3) $y = x$
- (4) $x + y = 4$

does not map the square back onto itself

6 The image of $\triangle ABC$ after a dilation of scale factor k centered at point A is $\triangle ADE$, as shown in the diagram below.



$$\triangle ABC \times k \rightarrow \triangle ADE$$

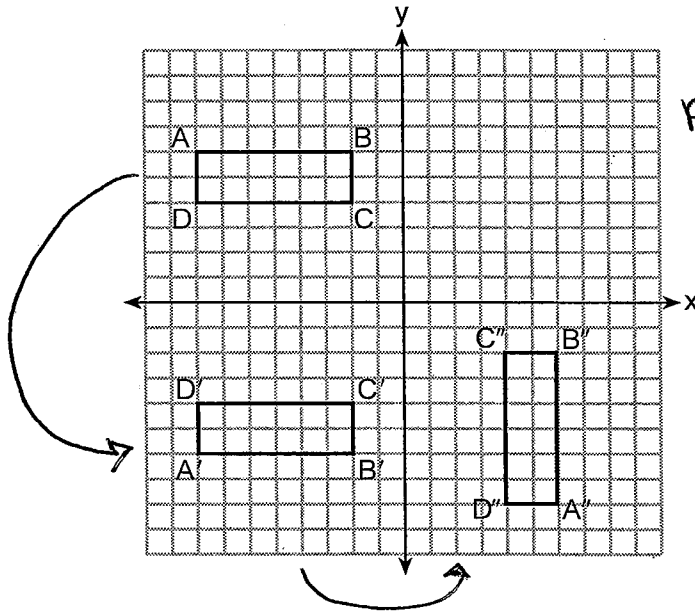
Which statement is always true?

- (1) $2AB = AD$
- (2) $\overline{AD} \perp \overline{DE}$
- (3) $AC = CE$
- (4) $\overline{BC} \parallel \overline{DE}$

We cannot say for sure choices 1, 2, and 3 b/c we DO NOT know the scale factor

Use this space for computations.

7 A sequence of transformations maps rectangle $ABCD$ onto rectangle $A''B''C''D''$, as shown in the diagram below.



$ABCD \rightarrow A'B'C'D'$
(reflection)

★ the vertices moved ★

$A'B'C'D' \rightarrow A''B''C''D''$
(rotation)

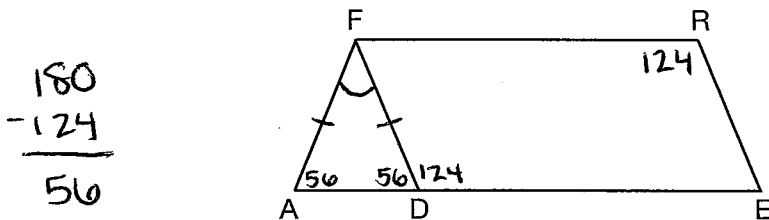
★ the rectangle is being turned ★

Which sequence of transformations maps $ABCD$ onto $A'B'C'D'$ and then maps $A'B'C'D'$ onto $A''B''C''D''$?

- (1) a reflection followed by a rotation
- (2) a reflection followed by a translation
- (3) a translation followed by a rotation
- (4) a translation followed by a reflection

8 In the diagram of parallelogram $FRED$ shown below, \overline{ED} is extended to A , and \overline{AF} is drawn such that $\overline{AF} \cong \overline{DF}$.

LABEL YOUR DIAGRAM!



180
- 56
- 56

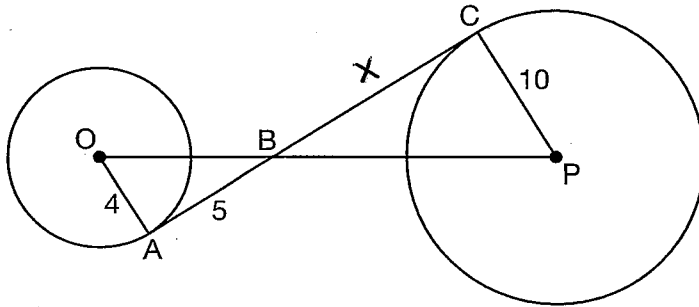
68

If $m\angle R = 124^\circ$, what is $m\angle AFD$?

- (1) 124°
- (2) 112°
- (3) 68°
- (4) 56°

Use this space for computations.

- 12 In the diagram shown below, \overline{AC} is tangent to circle O at A and to circle P at C , \overline{OP} intersects \overline{AC} at B , $OA = 4$, $AB = 5$, and $PC = 10$.



proportion!

What is the length of \overline{BC} ?

- (1) 6.4 (3) 12.5
 (2) 8 (4) 16

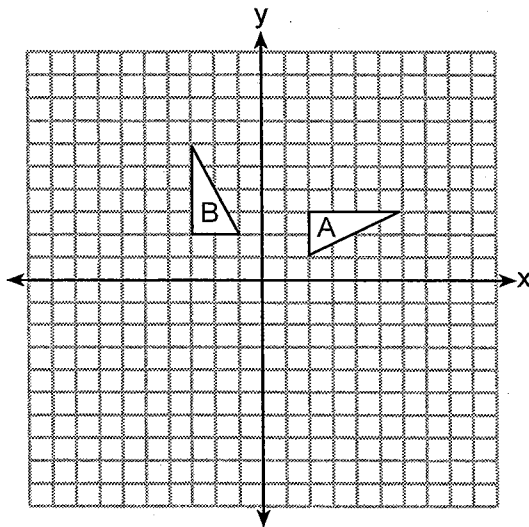
$$\frac{4}{5} = \frac{10}{x}$$

$$4x = 50$$

$$x = \frac{50}{4}$$

- 13 In the diagram below, which single transformation was used to map triangle A onto triangle B ?

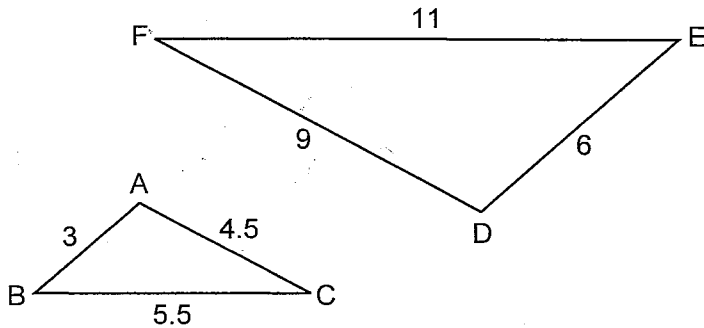
$$x = 12.5$$



- (1) line reflection (3) dilation
 (2) rotation (4) translation

Use this space for computations.

- 14 In the diagram below, $\triangle DEF$ is the image of $\triangle ABC$ after a clockwise rotation of 180° and a dilation where $AB = 3$, $BC = 5.5$, $AC = 4.5$, $DE = 6$, $FD = 9$, and $EF = 11$.



A \rightarrow D
 B \rightarrow E
 C \rightarrow F

Which relationship must always be true?

X (1) $\frac{m\angle A}{m\angle D} = \frac{1}{2}$

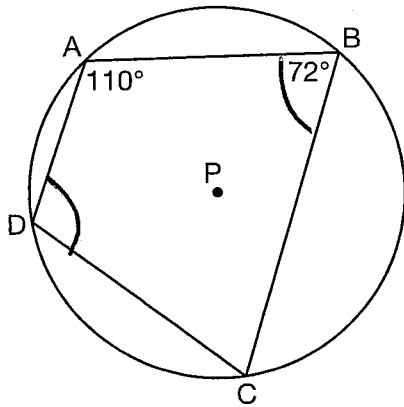
(3) $\frac{m\angle A}{m\angle C} = \frac{m\angle F}{m\angle D}$

X (2) $\frac{m\angle C}{m\angle F} = \frac{2}{1}$

(4) $\frac{m\angle B}{m\angle E} = \frac{m\angle C}{m\angle F}$

incorrect order in the second fraction

- 15 In the diagram below, quadrilateral $ABCD$ is inscribed in circle P .



When a quadrilateral is inscribed in a circle, the opposite angles of the quadrilateral add up to 180° (supplementary)

What is $m\angle ADC$?

(1) 70°

(2) 108°

(3) 72°

(4) 110°

$$\begin{array}{r} 180 \\ - 72 \\ \hline 108 \end{array}$$

half of a sphere

$$\text{radius} = \frac{1}{2} \text{diameter}$$

Use this space for computations.

$$r = \frac{1}{2}(10)$$

$$r = 5$$

$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \pi (5)^3$$

$$V = 523.5987756$$

- divide by 2 b/c we only want half of a sphere

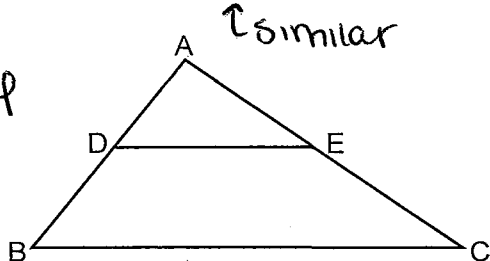
$$\rightarrow 523.5987756 \div 2 = 261.7993878$$

$$\begin{array}{r} \times 62.4 \\ \hline 16336.3 \end{array}$$

16 A hemispherical tank is filled with water and has a diameter of 10 feet. If water weighs 62.4 pounds per cubic foot, what is the total weight of the water in a full tank, to the nearest pound?

- (1) 16,336 (3) 130,690
 (2) 32,673 (4) 261,381

17 In the diagram below, $\triangle ABC \sim \triangle ADE$.



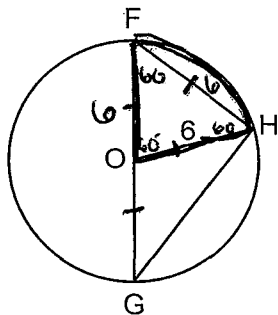
★ check all of your mc options ★

Which measurements are justified by this similarity?

- (1) $AD = 3, AB = 6, AE = 4, \text{ and } AC = 12$
 (2) $AD = 5, AB = 8, AE = 7, \text{ and } AC = 10$
 (3) $AD = 3, AB = 9, AE = 5, \text{ and } AC = 10$
 (4) $AD = 2, AB = 6, AE = 5, \text{ and } AC = 15$

$$\rightarrow \frac{2}{6} = \frac{5}{15} \rightarrow 30 = 30 \checkmark$$

18 Triangle FGH is inscribed in circle O , the length of radius \overline{OH} is 6, and $\overline{FH} \cong \overline{OG}$.



\overline{OH}
 \overline{OF}
 \overline{OG} } All radii
 All congruent

$\triangle FOH$ is an equilateral triangle. So, all the sides are \cong and all the angles are \cong . $180 \div 3 = 60^\circ$

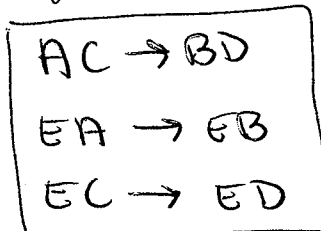
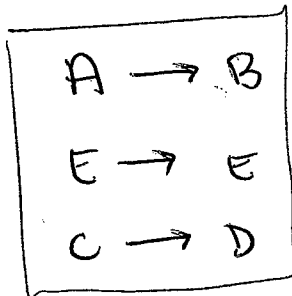
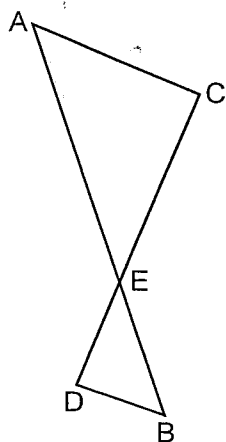
What is the area of the sector formed by angle FOH ?

- (1) 2π (3) 6π
 (2) $\frac{3}{2}\pi$ (4) 24π

$$\begin{aligned} \text{Area sector} &= \left(\frac{x}{360}\right) \pi r^2 \\ &= \left(\frac{60}{360}\right) \pi (6)^2 \\ &= 6\pi \end{aligned}$$

Use this space for computations.

19 As shown in the diagram below, \overline{AB} and \overline{CD} intersect at E , and $\overline{AC} \parallel \overline{BD}$.



Given $\triangle AEC \sim \triangle BED$, which equation is true?

X (1) $\frac{CE}{DE} = \frac{EB}{EA}$

X (3) $\frac{EC}{AE} = \frac{BE}{ED}$

(2) $\frac{AE}{BE} = \frac{AC}{BD}$

X (4) $\frac{ED}{EC} = \frac{AC}{BD}$

choices 1, 3, and 4
are all in the
incorrect order

20 A triangle is dilated by a scale factor of 3 with the center of dilation at the origin. Which statement is true?

(1) The area of the image is nine times the area of the original triangle.

(2) The perimeter of the image is nine times the perimeter of the original triangle.

(3) The slope of any side of the image is three times the slope of the corresponding side of the original triangle.

(4) The measure of each angle in the image is three times the measure of the corresponding angle of the original triangle.

Area $\sim r^2$ $r^2 = 3^2 = 9$

X Perimeter = 3 times

X slope doesn't change

X angle measure doesn't change

21 The Great Pyramid of Giza was constructed as a regular pyramid with a square base. It was built with an approximate volume of 2,592,276 cubic meters and a height of 146.5 meters. What was the length of one side of its base, to the nearest meter?

(1) 73

(3) 133

(2) 77

(4) 230

$V = \frac{1}{3} Bh$ → base area

$$2592276 = \frac{1}{3} B(146.5)$$

$$2592276 = 48.8B$$

$$\frac{2592276}{48.8} = \frac{48.8B}{48.8}$$

$\sqrt{53120.4098} = \sqrt{B}$ [OVER]

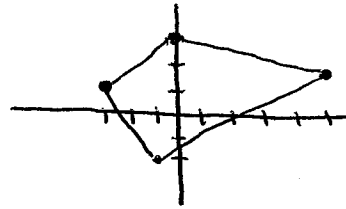
NOTE: The base is a square.
Base Area = $l \times w$ but $l = w$

When in doubt... sketch it out!

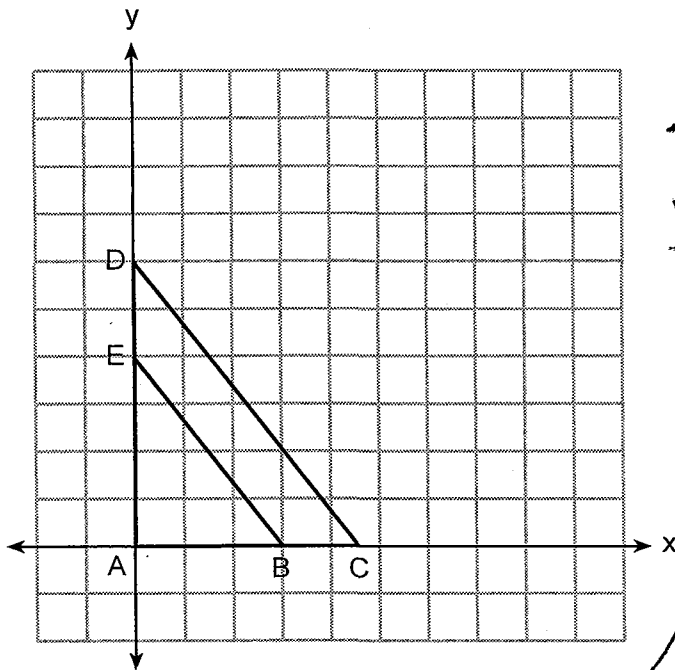
Use this space for computations.

22 A quadrilateral has vertices with coordinates $(-3,1)$, $(0,3)$, $(5,2)$, and $(-1,-2)$. Which type of quadrilateral is this?

- (1) rhombus (2) rectangle (3) square (4) trapezoid



23 In the diagram below, $\triangle ABE$ is the image of $\triangle ACD$ after a dilation centered at the origin. The coordinates of the vertices are $A(0,0)$, $B(3,0)$, $C(4.5,0)$, $D(0,6)$, and $E(0,4)$.



$$\triangle ABE \sim \triangle ACD$$

$$\frac{AB}{AC} = \frac{AE}{AD} = \frac{BE}{CD}$$

The ratio of the lengths of \overline{BE} to \overline{CD} is

- (1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{3}{4}$ (4) $\frac{4}{3}$

Since we can't count to find the length of \overline{BE} and \overline{CD} , we can use another pair of corresponding sides to find the ratio.

$$\frac{AB}{AC} = \frac{3}{4.5} = \frac{2}{3} \text{ (or) } \frac{AE}{AD} = \frac{4}{6} = \frac{2}{3}$$

24 Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at $(3,8)$. The line's image is

- (1) $y = 3x - 8$ (2) $y = 3x - 4$ (3) $y = 3x - 2$ (4) $y = 3x - 1$

* Since $(3,8)$ is on the line $y = 3x - 1$, the dilation is just the same line *

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 A wooden cube has an edge length of 6 centimeters and a mass of 137.8 grams. Determine the density of the cube, to the nearest thousandth.

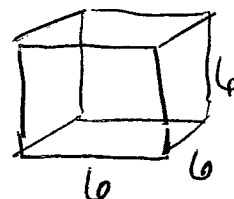
State which type of wood the cube is made of, using the density table below.

$$D = \frac{m}{V}$$



The unhappiest place on earth!

Type of Wood	Density (g/cm ³)
Pine	0.373
Hemlock	0.431
Elm	0.554
Birch	0.601
Ash	0.638
Maple	0.676
Oak	0.711



$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$



$$\text{Density} = \frac{137.8}{216}$$

$$= 0.638 \text{ g/cm}^3$$

Ash wood

• Volume = $l \times w \times h$

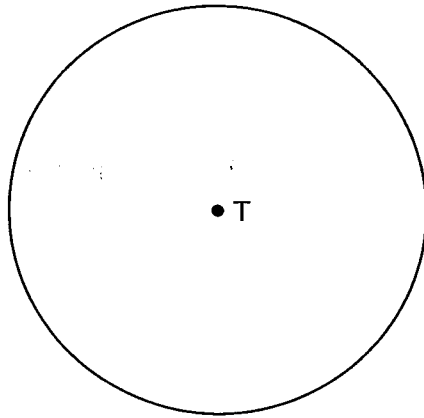
• Since this is a cube, $l = w = h = 6$

• $V = 6 \times 6 \times 6 = 216 \text{ cm}^3$

26 Construct an equilateral triangle inscribed in circle T shown below.

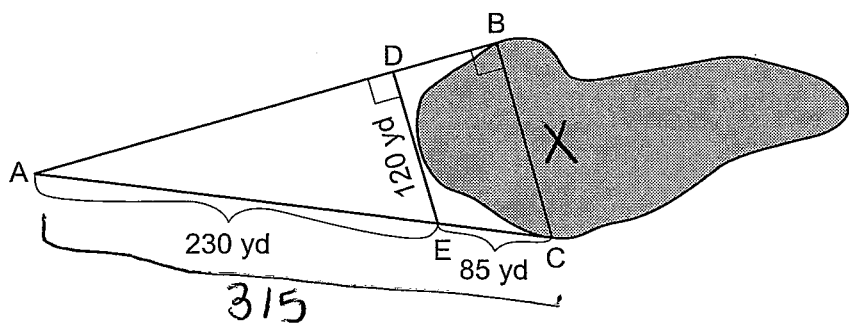
[Leave all construction marks.]

* Watch construction video on
youtube *



WeinsteinMAPMATH

27 To find the distance across a pond from point B to point C , a surveyor drew the diagram below. The measurements he made are indicated on his diagram.



Use the surveyor's information to determine and state the distance from point B to point C , to the nearest yard.

$\triangle ADE \sim \triangle ABC$ (by AA)

Set up a proportion!

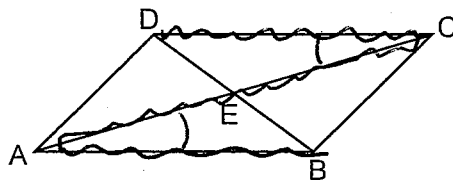
~~$$\frac{230}{120} = \frac{315}{X}$$~~

$$\frac{230X}{230} = \frac{37800}{230}$$

$$X = 164.3478261$$

$X = 164$ yds

28 In parallelogram $ABCD$ shown below, diagonals \overline{AC} and \overline{BD} intersect at E .

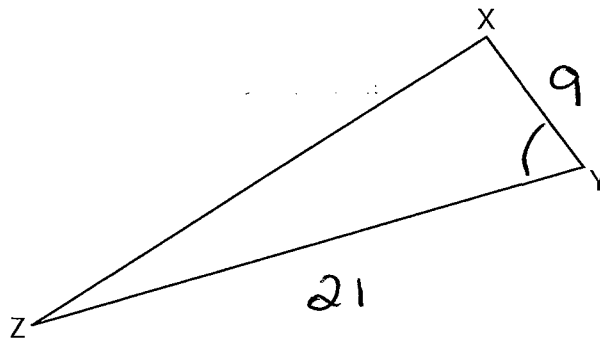
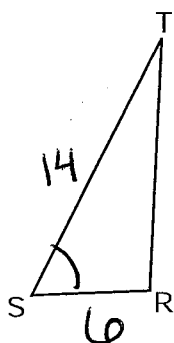


Prove: $\angle ACD \cong \angle CAB$

Statements	Reasons
① parallelogram $ABCD$, diagonals \overline{AC} and \overline{BD} intersect at E	① given
② $\overline{AB} \parallel \overline{DC}$	② opposite sides of a parallelogram are parallel
③ $\triangle ACD \cong \triangle CAB$	③ If 2 lines are parallel, then the alternate interior angles are congruent

LABEL YOUR DIAGRAMS!

29 Triangles RST and XYZ are drawn below. If $RS = 6$, $ST = 14$, $XY = 9$, $YZ = 21$, and $\angle S \cong \angle Y$, is $\triangle RST$ similar to $\triangle XYZ$? Justify your answer.



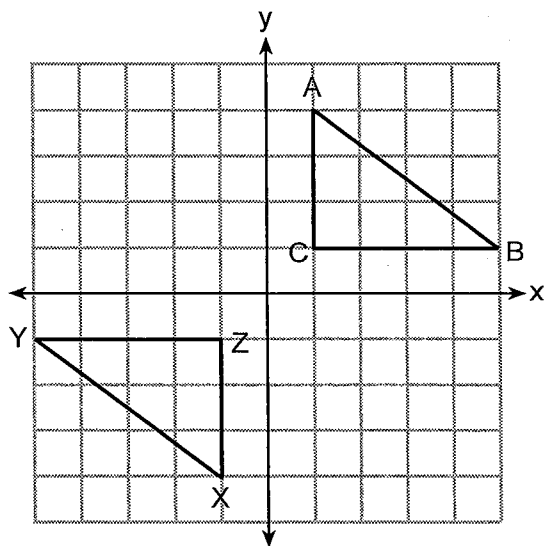
Set up a proportion and check if it works!

$$\frac{14}{6} = \frac{21}{9}$$

$$126 = 126 \checkmark$$

$\triangle RST \sim \triangle XYZ$ by SAS

30 In the diagram below, $\triangle ABC$ and $\triangle XYZ$ are graphed.



Use the properties of rigid motions to explain why $\triangle ABC \cong \triangle XYZ$.

The transformation is
a rotation. A rotation is
a rigid motion.

- or -

The transformation is a reflection
over the y-axis, followed by a
reflection over the x-axis. Reflections
are rigid motions.

31 The endpoints of \overline{DEF} are $D(1,4)$ and $F(16,14)$. Determine and state the coordinates of point E , if $DE:EF = 2:3$.

Step #1 : $2 + 3 = 5$ (# of sections the line segment is being broken into)

↓
This is where the $\frac{2}{5}$ comes from

X-distance

$$1 - 16 = -15$$

↓
15

* can't have a neg. distance *

x-coordinate

$$1 + \frac{2}{5}(15) = 7$$

↑

x-distance

y-distance

$$4 - 14 = -10$$

↓
10

y-coordinate

$$4 + \frac{2}{5}(10) = 8$$

↑

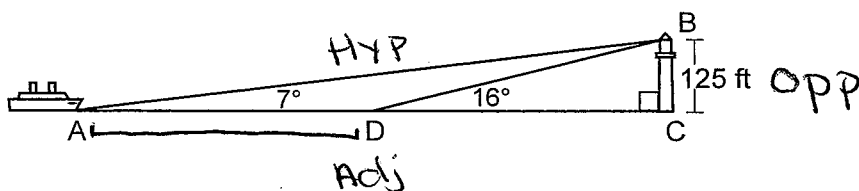
y-distance

E(7,8)

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 As shown in the diagram below, a ship is heading directly toward a lighthouse whose beacon is 125 feet above sea level. At the first sighting, point A, the angle of elevation from the ship to the light was 7° . A short time later, at point D, the angle of elevation was 16° .



To the nearest foot, determine and state how far the ship traveled from point A to point D.

① Find \overline{AC} SOHCAHTOA (using $\triangle ABC$)

- we have the opposite side
- we are looking for the adjacent side

$$\frac{\tan 7}{1} = \frac{125}{x} \rightarrow \frac{x \tan 7}{\tan 7} = \frac{125}{\tan 7} \rightarrow x = \frac{125}{\tan 7}$$

$$x = 1018.043$$

② Find \overline{DC} SOHCAHTOA (using $\triangle DBC$)

- we have the opposite side
- we are looking for the adjacent side

$$\frac{\tan 16}{1} = \frac{125}{x} \rightarrow \frac{x \tan 16}{\tan 16} = \frac{125}{\tan 16} \rightarrow x = \frac{125}{\tan 16}$$

$$x = 435.9268$$

③ Subtract! $1018.043 - 435.9268 = 582.1162$

$$= \boxed{582 \text{ ft}}$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

1 right angle means 2 ⊥ lines (neg. rec. slopes)

33 Triangle ABC has vertices with A(x,3), B(-3,-1), and C(-1,-4).

Determine and state a value of x that would make triangle ABC a right triangle. Justify why $\triangle ABC$ is a right triangle.

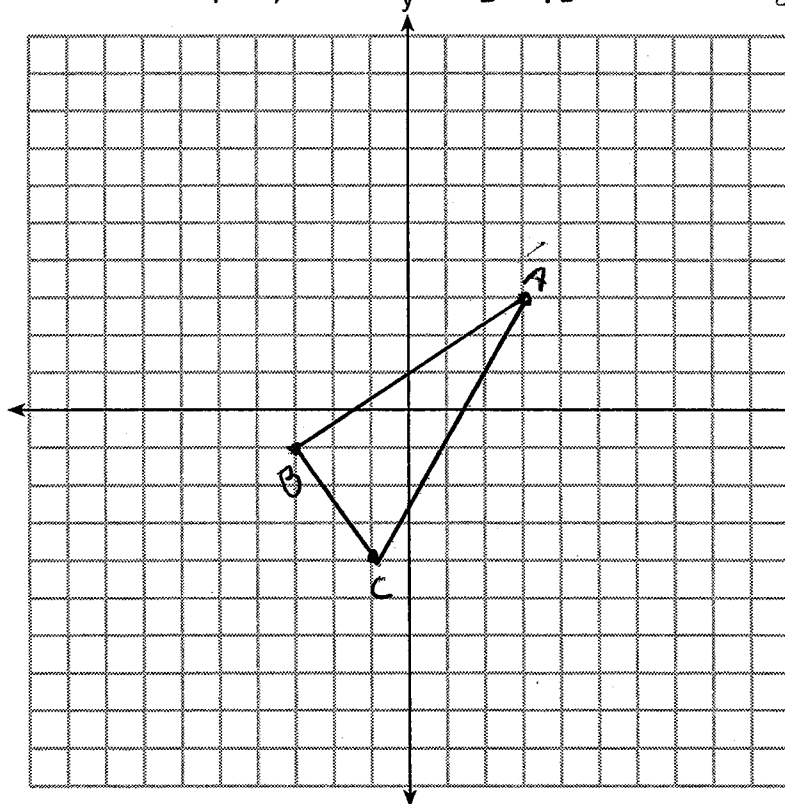
[The use of the set of axes below is optional.]

slope \overline{BC} B(-3, -1) C(-1, -4)
 x_1 y_1 x_2 y_2

$$\frac{-4 - (-1)}{-1 - (-3)} = \frac{-4 + 1}{-1 + 3} = \frac{-3}{2}$$

We want the slope of \overline{AB} to be $\frac{2}{3}$
 (b/c $-\frac{3}{2}$ and $\frac{2}{3}$ are negative reciprocals)

A(x, 3) B(-3, -1) → $\frac{-1 - 3}{-3 - x} = \frac{2}{3}$
 x_1 y_1 x_2 y_2



↓

$$\frac{-4}{-3-x} = \frac{2}{3}$$

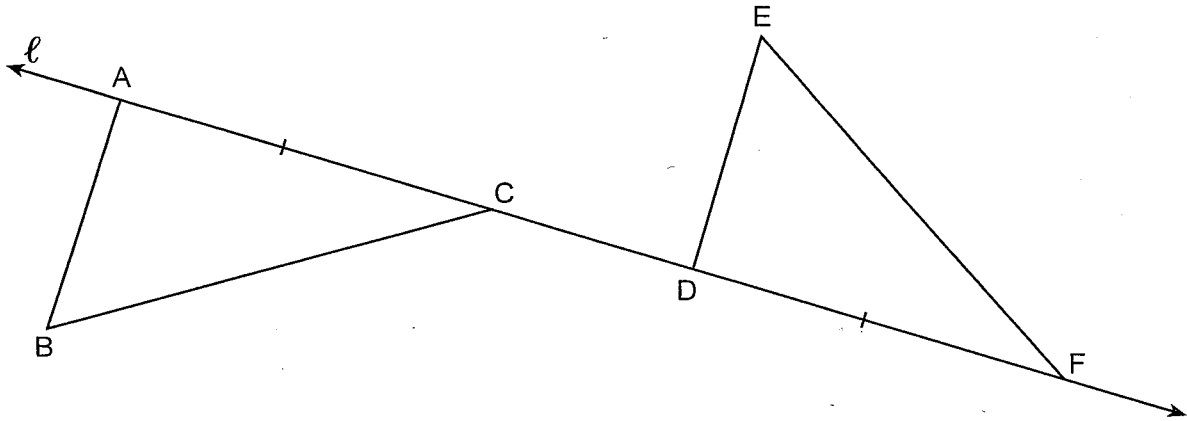
$$2(-3-x) = -12$$

$$-6 - 2x = -12$$

$$\begin{array}{r} -6 - 2x = -12 \\ +6 \quad +6 \\ \hline -2x = -6 \\ \frac{-2x}{-2} = \frac{-6}{-2} \end{array}$$

$x = 3$

34 In the diagram below, $\overline{AC} \cong \overline{DF}$ and points A , C , D , and F are collinear on line ℓ .



Let $\triangle D'E'F'$ be the image of $\triangle DEF$ after a translation along ℓ , such that point D is mapped onto point A . Determine and state the location of F' . Explain your answer.

$D \rightarrow A$
 $F \rightarrow C$

F' is located at point C
 b/c translations preserve distance.
 As $\triangle DEF$ is translated along
 line ℓ , point D is mapped to point A
 and point F is mapped to point C .

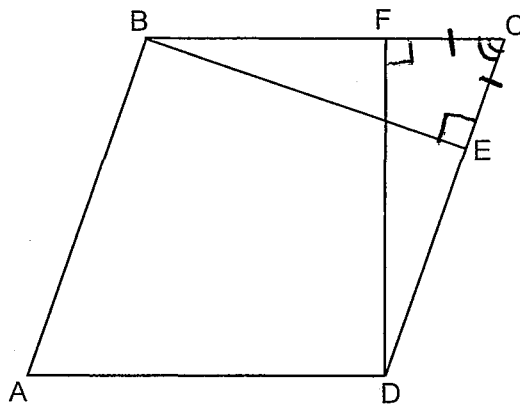
Let $\triangle D''E''F''$ be the image of $\triangle D'E'F'$ after a reflection across line ℓ . Suppose that E'' is located at B . Is $\triangle DEF$ congruent to $\triangle ABC$? Explain your answer.

Yes. As $\triangle DEF$ is translated
 along line ℓ to $\triangle D'E'F'$, size is
 preserved b/c a translation preserves
 size. As $\triangle D'E'F'$ is reflected over line ℓ
 and maps to $\triangle D''E''F''$, size is also
 preserved b/c a reflection is a rigid
 transformation

Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 In the diagram of parallelogram $ABCD$ below, $\overline{BE} \perp \overline{CD}$, $\overline{DF} \perp \overline{BC}$, and $\overline{CE} \cong \overline{CF}$.



Prove $ABCD$ is a rhombus.

Statements	Reasons
① parallelogram $ABCD$, $\overline{BE} \perp \overline{CD}$, $\overline{DF} \perp \overline{BC}$, $\overline{CE} \cong \overline{CF}$	① given
② $\angle BEC$ and $\angle DFC$ are right angles	② \perp lines form rt \angle 's
③ $\angle BEC \cong \angle DFC$	③ All right \angle 's are \cong
④ $\angle C \cong \angle C$	④ Reflexive
⑤ $\triangle BEC \cong \triangle DFC$	⑤ ASA \cong ASA
⑥ $\overline{BC} \cong \overline{DC}$	⑥ CPCTC
⑦ $ABCD$ is a rhombus	⑦ A parallelogram with consecutive \cong sides is a rhombus

$$\text{radius} = \frac{1}{2} \text{ diameter}$$

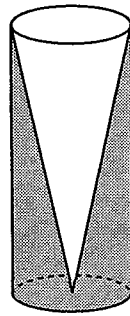
36 Walter wants to make 100 candles in the shape of a cone for his new candle business. The mold shown below will be used to make the candles. Each mold will have a height of 8 inches and a diameter of 3 inches. To the *nearest cubic inch*, what will be the total volume of 100 candles?

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi (1.5)^2 (8)$$

$$V = 6\pi$$

↑ volume of one candle



100 candles
 $6\pi \times 100 = 600\pi$
 $= 1885 \text{ in}^3$

Walter goes to a hobby store to buy the wax for his candles. The wax costs \$0.10 per ounce. If the weight of the wax is 0.52 ounce per cubic inch, how much will it cost Walter to buy the wax for 100 candles?

$$\begin{array}{r} 1885 \\ \times .52 \\ \hline \end{array}$$

980.2 ounces

$$\begin{array}{r} 980.2 \\ \times .10 \\ \hline \end{array}$$

\$98.02

If Walter spent a total of \$37.83 for the molds and charges \$1.95 for each candle, what is Walter's profit after selling 100 candles?

$$\begin{array}{r} \$1.95 \\ \times 100 \\ \hline \end{array}$$

$$\$195$$

↑
amount
of money
he made

$$\begin{array}{r} \$37.83 \text{ (molds)} \\ + \$98.02 \text{ (wax)} \\ \hline \end{array}$$

$$\$135.85$$

↑
amount of
money he
spent

PROFIT

$$\begin{array}{r} \$195 \\ - \$135.85 \\ \hline \end{array}$$

\$59.15